Review On Mathematical Models Used To Estimate Inner Temperature Variations Of Female Breast

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Abstract— *Breast cancer is a leading cause of mortality among women worldwide. Temperature-based techniques have emerged as a promising approach for breast cancer detection and prediction. This literature review aims to comprehensively analyse the existing research on mathematical models developed to predict the temperature gradient between the surface and core of the female breast. Various mathematical models, including Penne's bioheat transfer model, Wulff's model, Klinger's model, Chen and Holmes' model, and the porous media model, have been investigated. The strengths and limitations of each model, as well as their application in breast cancer risk prediction, have been examined. Additionally, the utilization of breast models, sensors, and validation techniques has been explored. The review highlights the need for further research to address the limitations of existing models and improve their accuracy in breast cancer diagnosis. The findings provide valuable insights for advancing temperature-based approaches and enhancing early detection strategies.*

Keywords— **Breast cancer, temperature-based techniques, mathematical models, bioheat transfer, temperature gradient, breast models, sensors.**

I. INTRODUCTION

Breast cancer remains the most prevalent cancer among women worldwide, with a significant impact on public health and individual lives. According to recent statistics, an estimated 2.3 million new cases were diagnosed in 2020, resulting in 684,996 deaths (Chen *et al.*, 2020). In Sri Lanka, the incidence of breast cancer has been rising rapidly over the past 15 years, with 4,447 cases identified in 2019 alone (Fernando *et al.*, 2018; Strategic Information Management Unit, 2021). Early detection is crucial for improving treatment outcomes and reducing mortality rates associated with breast cancer (Moreno and Herrera, 2019).

Breast cancer is a prevalent disease affecting women worldwide, with a significant number of new cases and deaths reported each year (Fernando *et al.*, 2018; Strategic Information Management Unit, 2021). Early detection plays a crucial role in improving the success rate of cancer treatment (Moreno and Herrera, 2019). In recent years, temperature-based techniques have shown promise for breast cancer detection and prediction (Kimberger *et al.*, 2009; Rassiwala *et al.*, 2014). Specifically, the temperature gradient between the surface and core of the breast has emerged as a valuable indicator for identifying abnormal tissue and potential malignancies (Lozano *et al.*, 2020). This has led to the development of mathematical models aimed at predicting the temperature variations within the breast and assessing their implications for breast cancer diagnosis.

The objective of this research paper is to provide a comprehensive literature review of the existing mathematical models that have been developed to predict the temperature gradient between the surface temperature and core temperature of the female breast. By examining the current state of knowledge in this field, we aim to identify the strengths, limitations, and opportunities for advancing temperature-based approaches for breast cancer diagnosis.

By gaining a deeper understanding of the existing mathematical models and their applications in predicting temperature variations within the breast, we can contribute to the development of more accurate and effective methods for early breast cancer detection. This has the potential to significantly impact patient outcomes and improve the overall management of breast cancer.

II. METHODOLOGY

The methodology adopted for this literature review encompassed a meticulous and systematic search and analysis of existing research studies that focused on the development of mathematical models to predict the temperature gradient in the female breast. Extensive exploration was conducted across multiple reputable scientific databases, including PubMed, IEEE Xplore, and Google Scholar. A careful selection of search terms, such as "breast cancer," "temperature-based techniques," "mathematical models," "bioheat transfer," and "breast temperature," was employed to ensure a comprehensive coverage of the relevant literature.

III. BIOHEAT MODELS USED TO ESTIMATE INNER TEMPERATURE VARIATIONS OF THE BREAST

Several bioheat models have been developed to predict the inner temperature of the breast and provide insights into tumour size and depth (Hristov, 2019). These models include Penne's bioheat transfer model, Wulff's model, Klinger's model, Chen and Holmes' model, and the porous media model.

A. Penne's bioheat transfer model.

This is widely used in bioheat transfer research, accounts for heat transfer through conduction, convection, and metabolism. However, it assumes steady-state conditions and neglects transient effects. Penne's equation is as follows.

$$\rho_t C_t \frac{\partial T_t(r,t)}{\partial t} = \frac{k_t}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_t\,(r,t)}{\partial r}\right] + \omega_b \rho_b C_b$$

$$(T_{a0} - T_t(r,t)) + Q_m$$

ρ_t - tissue density (kg/m^3)
C_t - tissue specific heat capacity (J/(kg·K))

$\frac{\partial T_t(r,t)}{\partial t}$ - partial derivative of tissue temperature Tt with respect to time t

k_t - tissue thermal conductivity (W/(m·K))
r - radial coordinate (m)

$\frac{\partial T_t\,(r,t)}{\partial r}$ - partial derivative of tissue temperature Tt with respect to the radial coordinate r

ω_b - blood perfusion rate (s^{-1})
ρ_b - blood density (kg/m^3)
C_b - blood specific heat capacity (J/(kg·K))
T_{a0} - arterial blood temperature (K)
Q_m - metabolic heat generation [W/m^3]

B. Wulff's model.

This is a modification of Penne's bioheat transfer model, incorporates metabolic heat generation within the tissue, providing a more accurate representation of heat generation in biological tissues. Similar to Penne's model, it assumes steady-state conditions and does not consider transient effects.

$$\rho_b C_p \frac{\partial T_t}{\partial t} = K_t \frac{\partial^2 T_t}{\partial x^2} - \rho_b v_h \left(c_p \frac{\partial T_b}{\partial x} - \Delta H_f \frac{\partial \varepsilon}{\partial x}\right)$$

ΔH_f - the specific enthalpy of the metabolic reaction.

$\rho_b v_h$ - the local blood mass flux

C. Klinger's model

This is a extends Penne's bioheat transfer model by incorporating blood perfusion heterogeneity within the tissue. By considering the spatial variation of blood perfusion rate, this model provides a more realistic representation of heat transfer within tissues. However, obtaining detailed knowledge of blood perfusion distribution may pose practical challenges.

$$\rho_t C_t \frac{\partial T_t}{\partial t} + \rho_b C_b V_b \,.\nabla T_t = k_t \nabla^2 T + Q$$

D. Chen and Holmes' model

This is another modification of Penne's bioheat transfer model, accounts for the temperature difference between arterial and venous blood to capture the convective heat exchange occurring in blood vessels. Although it improves the representation of heat transfer within the vasculature, it still assumes steady-state conditions and does not address transient effects.

Chen and Holmes' model in solid tissue space.

$$dV_s\left[\rho_s C_s \frac{\partial T_s}{\partial t}\right] = dQ_{ks} + dQ_{bs} + dQ_m$$

Chen and Holmes' model in vascular space.

$$dV_b\left[\rho_s C_s \frac{\partial T_b}{\partial t}\right] = dQ_{kb} - dQ_{bs} + \int_S (\rho_b C_b T) V d_s$$

E. Porous media model

In this model it treats the tissue as a porous medium, allowing for the analysis of heat transfer through interstitial fluid and the solid matrix. It considers convective heat transfer within the fluid phase and conductive heat transfer within the solid phase. This model offers insights into heat transfer mechanisms in tissues with complex structures or heterogeneous properties. However, it may require additional assumptions and parameters to accurately describe the properties of the porous medium.

Researchers have utilized these mathematical models to predict breast cancer risk and characteristics. The most widely used model is Penne's bioheat transfer equation (Korczak *et al.*, 2020; Paruch, 2020; Shrestha, Gurung and Gokul, 2021). In some cases, the Penne's bioheat equation has been used in inverse methods to determine tumour size and location (Hatwar and Herman, 2017). Additionally, various calculus concepts such as Laplace transform, ordinary differential equations, and partial differential equations have been incorporated into the development of mathematical models to analyse the thermal characteristics of the breast (Paruch, 2020), (Dolat Khan *et al.*, 2022), (Park and Yang, 2018). Some researchers have also employed the Stefan-Boltzmann equation to explore breast temperature characteristics (Souza *et al.*, 2015).

To model and validate these equations, researchers have developed breast models using tools such as COMSOL (Khomsi *et al.*, 2020), (Chanmugam, Hatwar and Herman, 2012), MATLAB, or finite element methods (Korczak *et al.*, 2020). Physical models, such as an artificial breast model comprising gelatine and silicon layers, have been commonly employed to validate mathematical models (Elouerghi *et al.*, 2022). Furthermore, porcine breast models (Donninger *et al.*, 2015) and Gonzalez-Hernandez breast models (Lozano *et al.*, 2020) have been utilized in some studies. Temperature data collection has been facilitated by the use of sensors, including bioheat sensors (Elouerghi *et al.*, 2022) and microwave sensors, which have proven to be accurate for breast cancer prediction (Wang, 2018).

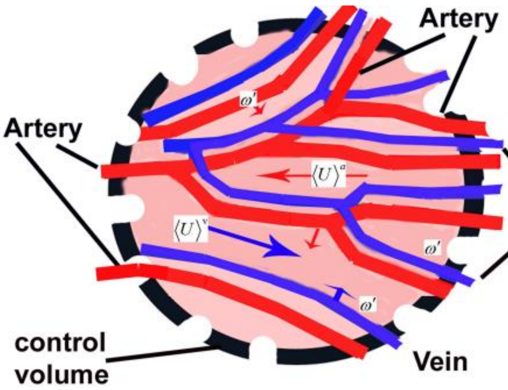

Figure 1: A schematic representation of a control volume V in a tissue with parallel blood supply via arteries and veins relevant to porous media studies(Hristov, 2019).

IV. PARTIAL DEFERENTIAL EQUATION THERMAL ANALYSIS ALGORITHM

The study(Park and Yang, 2018) uses a two-dimensional PDE thermal analysis model to develop a temperature distribution model for simulated breast cancer. It tracks changes in position and size, utilizing heat passing through defects, and investigates variations in surface temperature due to defect shifts. This innovative methodology overcomes limitations of conventional animal testing.

3D heat conduction is given by;

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

If k is a constant, then;

$$\left(\frac{\partial^2 T}{\partial x^2}\right) + \left(\frac{\partial^2 T}{\partial y}\right) + \left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = \frac{1}{\propto}\frac{\partial T}{\partial \tau}$$

Getting rid of the z-axis and the energy generated internally and using q(= 0) to enable two-dimensional analysis in Equation (1) ;

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c\frac{\partial T}{\partial \tau}$$

When converted to 2D;

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\propto}\frac{\partial T}{\partial \tau}$$

These boundary conditions were considered.

$$-k\frac{\partial T}{\partial x} = hf\,(T_s - T_f) + \sigma\varepsilon\,(T_s^4 - T_f^4)$$

where hf denotes the surrounding heat convection coefficient, Ts the material surface temperature, Tf the surrounding air temperature, is the Stefan-Boltzmann constant, and is the material surface radiation constant.

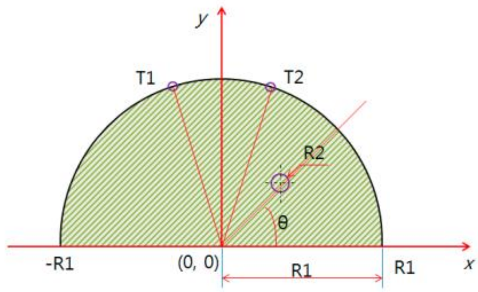

Figure 2: shows the thermal analysis model used in this study, where R1 represents the left and right coordinates of both endpoints of the test model, R2 represents the defect, and T1 and T2 represent the surrounding temperatures(Park and Yang, 2018).

V. MODEL FORMULATION USING FINITE ELEMENT METHOD

For numerical solutions, the finite element method is used in this study. The breast domain is divided into 862 triangular finite elements. The SST region's epidermal, dermal, and subcutaneous layers are divided into 128, 128, and 130 triangular finite elements, respectively. The glandular layer, tumor/cyst, and muscle with thoracic wall are each divided into 382 triangular finite elements.(Shrestha, Gurung and Gokul, 2021)

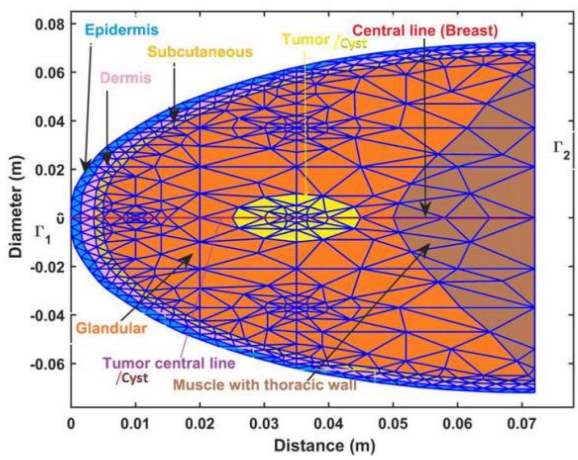

Figure 3: represents a schematic diagram of element-wise two-dimensional discretization of breast tissue containing a tumor/cyst. The tumor/cyst has a diameter of 20 mm and a center coordinate of (35, 0) (Shrestha, Gurung and Gokul, 2021).

Breast with hemi-spherical shape has five layers: epidermis, dermis, subcutaneous tissue, glandular layer, and muscle with thoracic wall. The X-axis is the breast's central line. The breast portion is symmetrical about the central line. A tumor/cyst is assumed in the glandular layer at the central line of the breast in the study. This is because most breast tumors/cysts develop in the glandular layer's lobules and milk ducts, and glandular is a medical term for breast.

VI. CONCLUSION

In conclusion, this literature review has provided a comprehensive analysis of mathematical models developed for predicting the temperature gradient in the female breast. The reviewed models, including Penne’s bioheat transfer model, Wulff's model, Klinger's model, Chen and Holmes' model, porous media model, PDE thermal analysis algorithm, and the model formulation using finite element method offer valuable insights into heat transfer mechanisms and temperature variations within breast tissue. However, it is essential to address the limitations of these models, such as their assumptions and neglect of transient effects, to improve their accuracy in breast cancer diagnosis. Future research should focus on refining these models and incorporating more realistic representations of the physiological processes involved. Additionally, the utilization of advanced techniques, including enhanced breast models and sensor technologies, should be explored to enhance the prediction capabilities of these mathematical models.

In essence, the synthesis of mathematical modelling, temperature-based techniques, and innovative methodologies holds significant potential for revolutionizing breast cancer diagnosis. As we advance our understanding of temperature variations within breast tissue, we pave the way for improved strategies in cancer detection, enhancing our ability to combat this widespread disease and transform patient outcomes. This research not only contributes to the scientific discourse surrounding breast cancer but also holds the promise of driving tangible progress in clinical practice and patient care.

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