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A Comparison of Nonlinear Material Models Used in Pavement Response Modelling

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Abstract: The numerical modelling of pavement responses is very much influenced by structural properties of pavement. The structural properties are modelled by material models. Clear understandings of pavement responses with respect to the material models are necessary to evaluate the accuracy of prediction. This paper is aimed at investigating the application of an inverse modelling technique to find the material parameters of three different models using Repeated Load Triaxial test data. The method of estimating material parameter values is based on least squares technique. Computations were performed using ABAQUS in MATLAB platform. The results of the computations are compared to each other. Numerical simulations and comparisons show a good agreement between estimated values (modulus, strain, and stress deflection) obtained using three different models.

Keywords: Pavement response; Finite element modelling; USDFLD; UHYPEL.

Introduction

Permanent deformation of pavement decreases its durability and results in serious traffic safety problems. This problem will become more serious with increase in axle loading and repetitions. Therefore, optimal cost and serviceability estimation of new pavement design problem have to be studied carefully. This can be done efficiently using numerical modelling of pavement response. The accuracy associated with modelling pavement response is highly dependent on material model. Inaccuracy in the model can lead to differences between calculated and actual estimation.

Numerical modelling of pavement response may be used during a design process to build the pavements. The pavements response simulation describes the mechanical response of material under axial loads. Several deformation models are available in the literature to study and control permanent deformation in flexible pavements. They use different material models and it depends on stress level. This research evaluates two of the most recent models with the most popular $K-\theta$ model for suitability in the prediction of rutting potential of asphalt mixes.

Material Models

Granular materials make up a discontinuous particulate medium physically and its resilient performance is strongly influenced by the applied wheel load levels and the thicknesses of surface materials overlain. The resilient behaviour of granular materials defined by resilient modulus is influenced by stress level, density, grain size, aggregate type, particle shape, moisture content, and number of load applications. Resilient models of granular materials increase with increasing stress states. There are several mathematical models have been developed using different stress components.

One of the most popular model was developed by Hicks and Monismith [4]. This model, known as the $K-\theta$ model, has been the most widely used for material modeling as a function of stress state applicable to granular materials. it is given by

$$M_r = K\theta^n. \tag{1}$$

where $\theta = \sigma_1 + \sigma_2 + \sigma_3$, σ_1 σ_2 σ_3 are principal stress, $\boldsymbol{K}$, $\boldsymbol{n}$ are material constants.

Then in late eighties researchers found that the $K - \theta$ model was not sufficient to describe the shear behaviour and made some modifications to the model. This model was proposed by Uzan [8]. This was the model used by Bruce Steven [2], [3] in his PhD thesis to develop a nonlinear pavement response and performance model for calibration and verification of two thin surfaced unbound granular pavements. It is given by

$$M_r = k_1 p_a \left(\frac{\theta}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1\right) \tag{2}$$

where

$$\begin{aligned} \theta &= \sigma_1 + \sigma_2 + \sigma_3 \\ \tau_{oct} &= \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \\ p_a &= 100\ kPa \end{aligned}$$

M_r resilient modulus, k_1, k_2, k_3 are constants, σ_1, σ_2 and σ_3 are principal stress. This model will be referred in this paper as the Bruce Steven's model. This model was used to construct granular layers in our previous research works [5], [6], [7].

Werkmeister[9] proposed the nonlinear 'Dresden Model' in her PhD thesis. This nonlinear elastic model is expressed in terms of resilient modulus, E, and Poisson ratio, υ, as follows:

$$\begin{aligned} E &= p_a \left(q + C\left(\frac{\sigma_3}{p_a}\right)^{q_1}\right)\left(\frac{\sigma_1}{p_a}\right)^{q_2} + D \quad (3) \\ \upsilon &= R\frac{\sigma_1}{\sigma_3} + A\frac{\sigma_1}{p_a} + B \quad (4) \end{aligned}$$

where σ_3(kPa)) minor principal stress (absolute value); σ_1 (kPa) major principal stress (absolute value); D (kPa) constant in term of modulus of elasticity; p_a = 1 kPa, q, C, q_1, q_2, R, A and B are model parameters.

Inverse Modelling

Inverse modelling is the estimation of model constants from data. It is a discipline that provides tools for the efficient use of data in the estimation of constants appearing in the models. In our problem, structures of equation are known; measurements of modulus for various different principal stresses are obtained from Repeated Load Tri-axial (RLT) test and given in Table 1. Some of the constants are unknown.

Table 1: RLT test results

Major stress (kPa)	Minor stress (kPa)	Modulus (MPa)
210	120	487
167	66.7	352
142	41.7	289
270	90	446
470	140	585
530	110	549

The goal of this section is to find the optimal values of the constants appearing in Equations 1-4 from the available measurements given in Table 1. Taking natural logarithms of both sides of Equations 1 and 2 gives respectively

$$\ln M_r = n \ln \theta + \ln K \tag{5}$$

$$\ln M_r = \ln(k_1 p_a) + k_2 \ln\left(\frac{\theta}{p_a}\right) + k_3 \ln\left(\frac{\tau_{oct}}{p_a} + 1\right) \tag{6}$$

If the data values are transformed by letting:

$$W = \ln M_r,\ x = \ln\theta,\ X_1 = \ln\left(\frac{\theta}{p_a}\right) \text{ and } X_2 = \ln\left(\frac{\tau_{oct}}{p_a} + 1\right)$$

then Equations 5, 6 for p data values respectively become

$$\begin{bmatrix} \ln M_1 \\ \ln M_2 \\ \vdots \\ \ln M_p \end{bmatrix} = \begin{bmatrix} \ln \theta_1 & 1 \\ \ln \theta_2 & 1 \\ \vdots & \vdots \\ \ln \theta_p & 1 \end{bmatrix} \begin{bmatrix} n \\ \ln K \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} \ln M_1 \\ \ln M_2 \\ \vdots \\ \ln M_p \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} \\ 1 & X_{1,2} & X_{22} \\ \vdots & \vdots & \vdots \\ 1 & X_{1p} & X_{2p} \end{bmatrix} \begin{bmatrix} \ln(k_1 p_a) \\ k_2 \\ k_3 \end{bmatrix} \quad (8)$$

The step then is to estimate n, K using linear regression and k_1, k_2, k_3 using multiple linear regression.

But the Equation (3) cannot be linearized easily and therefore we then relied exclusively on MATLAB s routine *lsqnonlin* to find the constants appearing in Equation (3). For p data values Equation (3) can be written as

$$\mathbf{E} = f(\mathbf{P}) \quad (9)$$

where $E = [E_1\ E_2, \cdots E_p]$, $P = [q, C, q_1, q_2, D]$ and $\boldsymbol{f}$ is a function. To estimate $\boldsymbol{P}$ following minimization problem is solved.

$$\text{minimize} \quad \mathbf{Z} = \| f(\mathbf{P}) - \mathbf{E} \|_2^2, \quad (10)$$

where $f(\mathbf{P})$, $\mathbf{E}$ are vectors containing estimated and measured modulus values respectively and $\mathbf{P}$ is the vector of unknown values.

All of the above mentioned models except the Sabine's models uses constant poisson ratio. We therefore investigated the effect of assuming constant poisson ratio on Sabine's model. For this investigation we simulate the pavement response model with Sabine's Equation (3) for the poisson rations 0.2, 0, 3 and 0.4. Results of the comparison are summarized in Figure 1. Figures A, B, C and D respectively illustrates variation of modulus, vertical strain, vertical stress and vertical displacement along the vertical axis under the center of the wheel. These results suggest that the effect of keeping constant poisson ratio on modulus, stress, strain and displacement are minimal.

Listed in Tables 2, 3 and 4 are the estimates of constant values for the $\boldsymbol{K - \theta}$, Bruce and Sabine's models for the data value given in Table 1 and poisson ratio of 0.35.

Table 2: $K - \theta$ model

k	n
343860	0.55

Table 3: Uzan's model

k_1	k_2	k_3
1790	0.7458	-0.3455

Table 4: Sabine's Model

Q	C	q_1	q_2	D
14004	6540	0.35	0.33	$65\text{x}10^6$

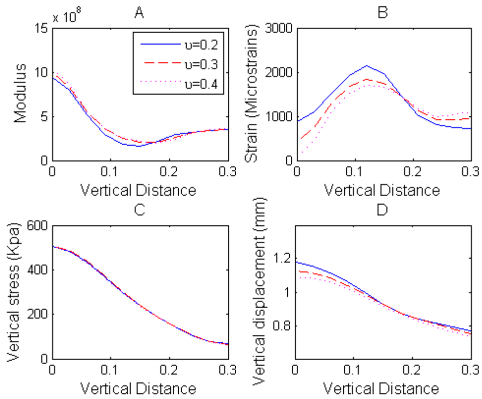


Figure 1: Effect of Poisson Ratio

Finite Element Model

The general purpose finite element program ABAQUS is used in this investigation to simulate the deformation process as shown in Figure 2. By making use of symmetry in the geometry we only considered the quarter of the model. FEM model contain only two layers. The top layer is 300 mm height and modulus of elasticity is modeled by the equation (3) and the poisson ratio of 0.35. The bottom layer has a height of 1200 mm with modulus of elasticity of 24 MPa and a posson ratio of 0.4. The length and width of FEM section is 750 mm.

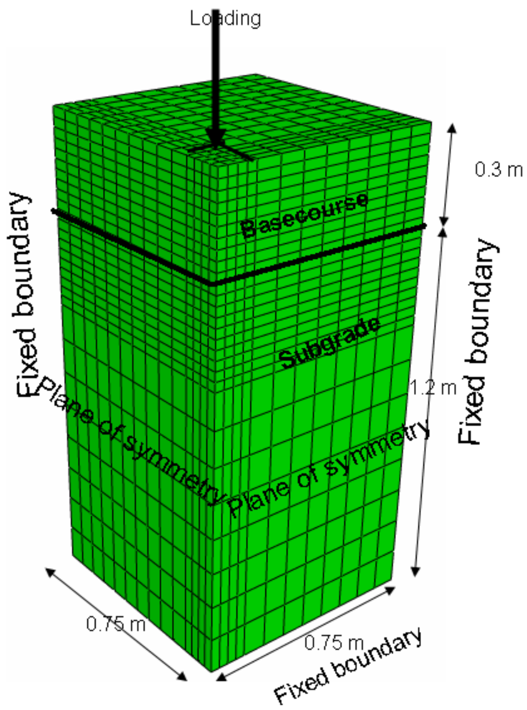


Figure 2: FEM Model

Subroutine

The material model described in Equation (1), (2) and (3) are implemented in ABAQUS using the user-written subroutine USDFLD and UHYPEL. Figure 3 illustrates the structure of the user defined subroutine. The most simple usage of this routine defines modulus in terms of strain but in our case modulus is a function of stresses. Stress are brought into the UHYPEL subroutine via the use of user predefined fields.

The subroutine USDFLD allows the user to define field variables at every material point of an element and provides access to a number of variables such as stress, strains. A utility subroutine GETVRM is used to access these variables at every material point in the calculation domain. The subroutine USDFLD is used to calculate the absolute values of principal stresses and passed into the subroutine UHYPEL.

Results and Discussion

In this section, we present numerical calculations to evaluate the differences between model predictions. To do so we consider the pavement comprised a 300 *mm* thick layer of granular material placed directly on top of a 1200 *mm* thick sub base. The length and width of the loaded area in the quarter 3D model is 124 *mm* and the contact stress is 0.510 Nmm^{-2}. The results of the simulation using three different material models are given in Figures 4, 5, 6 and 7.

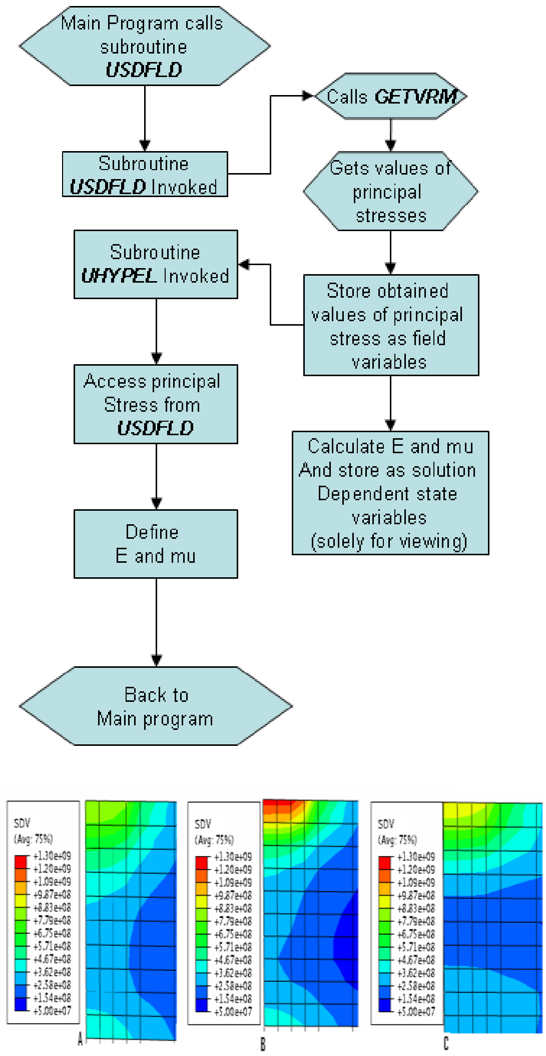


Figure 3: USDFLD.

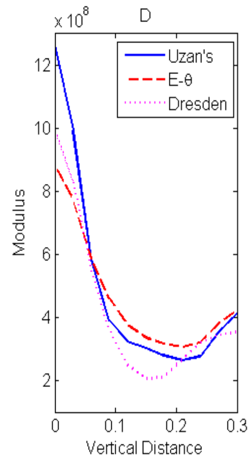
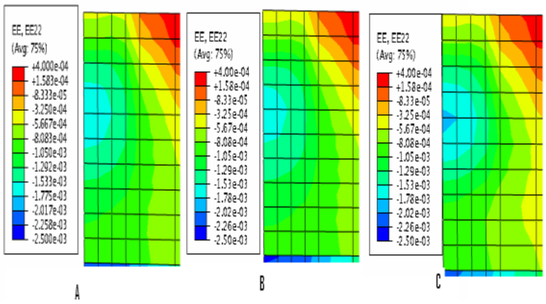


Figure 4: Modulus: (A)E□_ model, (B) Uzan's model, (C)Sabine's model, (D) Modulus at the center, (E) Modulus at the edge.



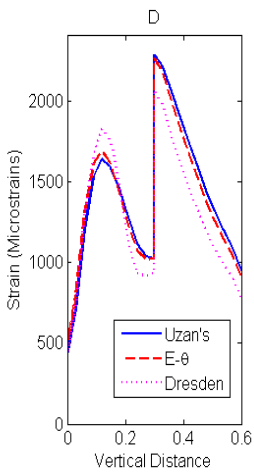
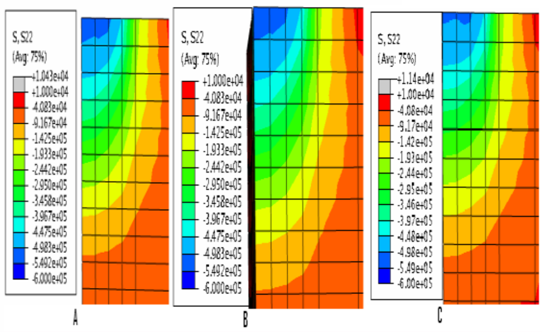


Figure 5: Vertical strain: (A)E □ _ model, (B) Uzan's model, (C)Sabine's model.



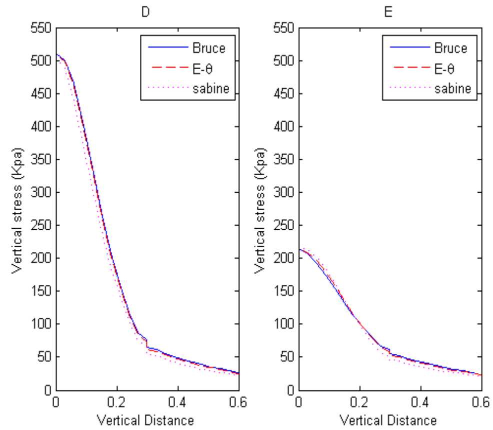


Figure 6: Vertical stress: (A)$E - \theta$ model, (B) Bruce's model, (C)Sabine's model, D) Modulus at the center, (E) Modulus at the edge.

Figures 4 A, B and C show the contour plot of modulus under load contact area for three models respectively. The modulus variations along the vertical axis under the centre of load contact area and at the edge of the load contact area are given in Figures 4D and 4E respectively. It can be seen that results are very close except near the top surface and near the middle of the base course.

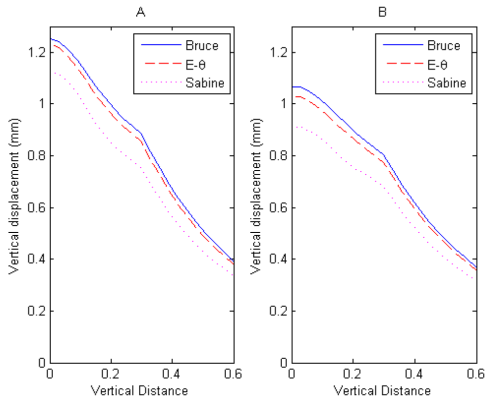


Figure 7: Vertical displacement: (A) at the center, (B) at the edge.

Figures 5 A, B and C show the contour plot of vertical strain under load contact area for three models respectively. The vertical strain variations along the vertical axis under the centre of load contact area and at the edge of the load contact area are given in Figures 5D and 5E respectively. It can be seen that results are very close except the magnitude of the peaks. Sabine's model slightly over estimated the strain values near the centre of the base course material and under estimated near the bottom of the base course. The strain values inside the sub base also influenced by the strain values near the bottom of the base course layer.

Figures 6 and 7 respectively shows the vertical stress and displacement under load. Stress values are very close. The vertical distance and magnitude of stress values are well matched.

Summary and Discussion

The intention of this paper is to demonstrate an inverse model capable of concurrently estimating the material constants of (1) $E - \Theta$, (2) Uzan's and (3) Sabine's material models using the RLT experimental data. The approach is based on a least squares estimation using simulated strain value and coupled with Finite element technique.

A finite element model was developed using ABAQUS to predict the deformation behaviour of flexible pavement structure subjected to a static and dynamic loads. The results obtained using three different material models mentioned above were compared. A comparison between the results suggest that results obtained using both models are nearly same.

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