Comparison of AIC, BIC and AICC in Autoregressive Order 1 Time Series Models

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Abstract - Selecting a suitable model for a given data set from a set of competing models is known as statistical model selection. There are a number of model selection criteria such as Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC) and Corrected Akaike's Information Criterion (AICC). Even though it is crucial to choose a model that best fits a particular set of data, still there is no proper way to resolve this question. So, it is needed to find evidence to determine what is the best model selection criterion. The objective is to compare the most commonly used penalized model selection criteria namely AIC, BIC and AICC in the field of time series to determine the best performance criteria. This study mainly focus on Autoregressive models of order 1 ($X_t = \emptyset X_{t-1} + Z_t$ where $Z_t \sim N(0,1)$) and investigate the performance of AIC, BIC and AICC for different scenarios.

The behavior of these model selection criteria is examined and compared through Monte Carlo simulations. Here 500 series were simulated considering an Autoregressive model of order 1 for different sample sizes and different autoregressive coefficient values. The results of the simulation study are shown in bar graphs with the percentages of accurate selections and compared the performance of AIC, BIC and AICC. The study concludes that mostly BIC outperforms AIC and AICC for the considered scenarios.

Keywords: AIC, BIC, AICC, time series, autoregression

Extended Abstract –

Introduction

Time series data is sequence of time ordered data. Time series data can be implied through real life examples such as hourly readings of air temperature, monthly rainfall data, continuous monitoring of a person's heart rate, daily closing price of a company stock etc. To analyze the behavior of such series it is vital to fit accurate model to observe series. So, model selection and model selection criteria are prominent members in time series. Various model selection criteria are used in researchers such as Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), Corrected Akaike's Information Criterion (AICC), Minimum Description Length (MDL), Deviance Information Criteria (DIC), Focused Information Criterion (FIC) etc. (Brockwell and Davis, 1991)

The core objective of this study is to compare the three main model selection criteria AIC, BIC and AICC and to determine what is the best model selection criteria on a time series among them through a simulation study. There is no proper evidence to say what is the best model selection criteria among AIC, BIC and AICC. So often statisticians struggle in selecting the best model selection criterion. To investigate the performance of these information criteria they will be applied on time series data to compare and select the best among them. Even though current various studies have used the comparison of AIC, BIC and AICC in selection of various relationships, there is no proper literature can be found relating to time series models. Therefore, those who are willing to study more on model selection criteria on time series models will be able to get a proper idea by following this article.

Methodology

The performance of the model selection criteria was assessed though a simulation study. These simulation routines were carried out using the statistical software R. The simulations were performed for $AR(1)$. In the first stage data were generated for a sample size 100 and fixing the other parameter values as $\sigma^2 = 1$ and $\emptyset = 0.5$. The process was performed for 500 times. Then the lowest AIC, BIC an AICC values for each 500 series were generated using R codes. Then obtained the percentages of times that each criterion selects the true model $(AR(1))$. Afterwards change the parameter $\emptyset$ value into $-0.1, -0.5, -0.7, -0.9, 0.1, 0.7, 0.9$ where the $|\emptyset| < 1$, then did the same procedure for sample size (n) $= 12, 50, 200, 365, 500$ and 1000 and there also obtained the percentage of times that each criterion selects the true model.

Results and Discussion

This work aimed to review and investigate the performance of AIC, BIC and AICC information criteria to determine what is the best model selection criteria on $AR(1)$ model. The performance of these criteria was evaluated through Monte Carlo simulations for different sample sizes and for different parameter values of $AR(1)$ model.

Let z_t be a time series,

$AR(1)$ model can be defined as follows,

$$z_t = \emptyset z_{t-1} + a_t \qquad where \ |\emptyset| < 1, \qquad a_t \sim NID(0,1)$$

Here, a_t is an independent and identically distributed White Noise time series with mean zero and variance is 1.

The following Table 1 summarizes the simulation cases that carried out in the study.

Table 1. Summary Table about the simulation study

Case	Sample Size	Parameter Value $|\emptyset|$
Case I	$n = 12$	0.1, 0.5, 0.7, 0.9
Case II	$n = 50$	0.1, 0.5, 0.7, 0.9
Case III	$n = 100$	0.1, 0.5, 0.7, 0.9
Case IV	$n = 200$	0.1, 0.5, 0.7, 0.9
Case V	$n = 365$	0.1, 0.5, 0.7, 0.9
Case VI	$n = 500$	0.1, 0.5, 0.7, 0.9
Case VII	$n = 1000$	0.1, 0.5, 0.7, 0.9

Although the possible combinations are infinite, we have limited our study to above mentioned categories. The reason behind why we have chosen the above-mentioned values for $\emptyset$ is because they represent weak, moderate and strong auto correlation. Since AIC, BIC and AICC depend on the maximum likelihood estimators and the maximum likelihood estimators have good asymptotic properties, therefore, to see how AIC, BIC and AICC behave with small, moderate and large sample sizes we have considered reasonable sample sizes in order to get good assurance regarding the final conclusions.

For each case the performance of each criterion is given by bar charts giving the percentage of times that the AIC, BIC and AICC selected the correct model.

Here for this article simulation Case I with $\emptyset = -0.1$ is elaborated.
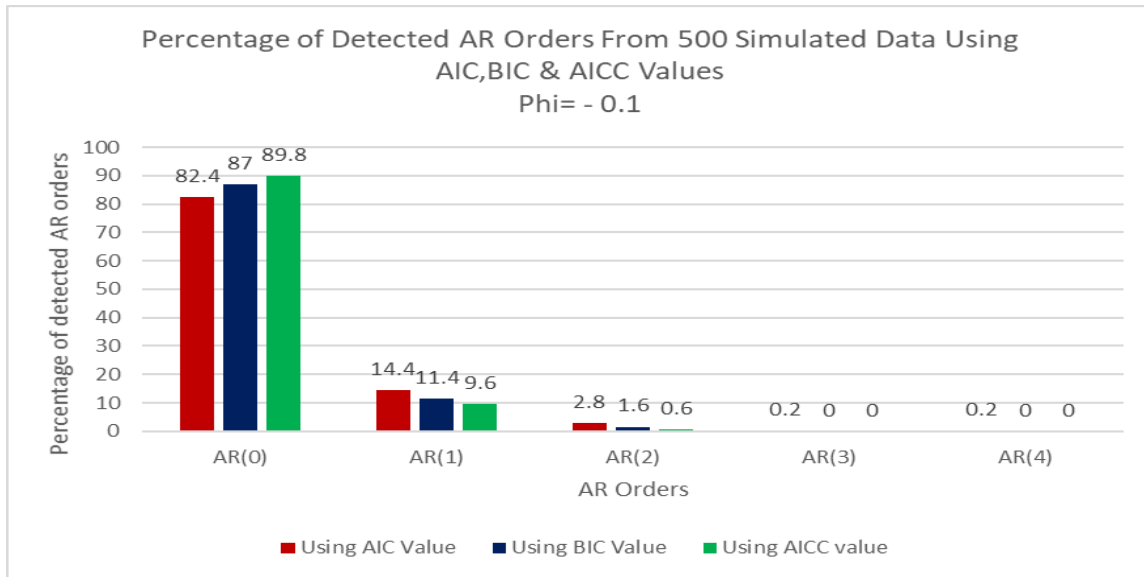
E.g.: Case I : ⍰ = 12

⍰h⍰ (∅) = -0.1

Figure 1. Percentage of detected AR orders when n=12 and phi=-0.1

According to the above Figure 1 almost all three information criteria: AIC, BIC and AICC are fail in selecting the true model ⍰⍰(1) considering their minimum values. Here AIC, BIC and AICC all prefer for ⍰⍰(0) mostly.

Here the ∅ value closes to zero, it represents the weak autocorrelation. Therefore, because of the less autocorrelation all the three information criteria fail in detecting the true model.

Likewise, for the case ⍰ = 12 the parameter ∅ value was changed to -0.5, -0.7, -0.9, 0.1, 0.5, 0.7,0.9 and detected the performance.

Then all the other cases were carried out by changing the sample size and also the parameter ∅ value.

The results obtained from the simulation study have been demonstrated in the below Table 2.

Table 2. Summary Table on the selected criterion

The criterion which selects the AR (1) correctly	n=12	n=50	n=100	n=200	n=365	n=500	n=1000
$|\emptyset| = 0.1$	All three criteria fail to select AR (1)	All three criteria fail to select AR (1)	All three criteria fail to select AR (1)	All three criteria fail to select AR (1)	AIC and AICC	AIC and AICC	AIC and AICC
$|\emptyset| = 0.5$	AIC	BIC	BIC	BIC	BIC	BIC	BIC
$|\emptyset| = 0.7$	AICC	BIC	BIC	BIC	BIC	AIC and AICC	BIC
$|\emptyset| = 0.9$	AICC	BIC	BIC	BIC	BIC	AIC and AICC	BIC

It was found that when the $|\emptyset|$ gets closer to '1' AIC, BIC and AICC tend to detect the true model $AR(1)$ because of the presence of the higher autocorrelation. When $|\emptyset|$ gets closer to '0' AIC, BIC and AICC fail to detect the true model because of the less auto correlation.

In the case of very small sample size (namely $n = 12$) it was found that AIC and AICC works extremely well rather than BIC. For small sample size (namely $n = 50$) and for immediate sample sizes (namely $n = 100, 200$ and 365) BIC tends to perform better. In the case of large sample sizes (namely $n = 500$) the percentages of AIC and AICC in detecting $AR(1)$ were greater than the percentage obtained for BIC. Therefore, in this case AIC and AICC outperform BIC. For very large sample size (namely $n = 1000$) BIC is better in selecting the true model $AR(1)$.

Considering both the sample size and the parameter values the overall study clearly reveals that for BIC the percentage of correct answers stabilized at approximately in between 60% to 98%. All in all, our study concludes that mostly BIC outperforms AIC and AICC.

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