Box-Jenkins Approach to Forecast Monthly Gold Price in Sri Lanka

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Abstract: *Depending on recent records, the price of gold has been rising day by day more than the past; similar characteristics has been shown in Sri Lankan gold price as well. What would be the causes behind this? Basically, this study has been carried out on two levels. For first level, we constructed models to forecast monthly gold price. The data was mined form World Gold Council and Central Bank of Sri Lanka. The sample data of gold price were gathered from 2007 January to 2016 March in the currency of US dollars per troy ounce. It was converted into Sri Lankan rupees per 22 carats. 75% of data were used to build the model and remaining data were used to forecast the gold price and to check the accuracy of the model. Box-Jenkins, Auto Regressive Integrated Moving Average (ARIMA) methodology has been used to build forecasting models.*

Keywords— ARIMA, Gold price, ADF test,

1. INTRODUCTION

Identifying the movements of the world economy remains uncertainty in many trades financial policy making process is very essential. Further foreign investors are not interested in investing on local stocks when countries currency is depreciating, as it would be a reason to diminish their return on invested assets. In such situations investors try to diversify their investments across multiple portfolios such as precious metal, bonds etc. As a result, they tend to shift from high risk instruments to less risk instruments with the intention of minimizing the loss. Commodities are among these instruments that may protect themselves from particular risks. Therefore, precious metals such as gold and silver and gems are popular among investors in past decades. Generally, there are many factors such as company performance, dividends, Inflation rate, interest rate, exchange rate, gold rate etc.. The last several years have seen a rapid rise in the demand of gold. Sectors where the gold has been used is expanded in different areas such as medicine, engineering and environmental management. According to the World Gold Council (WGC) records the annual volume of gold bought by investors has increased by "at least 235% over the last three decades. Not only that gold price has increase in several levels [4]. In order to that Sri Lankan gold price has shown remarkable growth in present (see www.ideabeam.com). Therefore, it is highly demand and standard metal among others. Sri Lanka has a long traditional association with the international Gem and jewelry trade and identified as a significant resource place for gold. Over past few decades Sri Lanka gold shows a remarkable position in the international gold market in parallel to local market; as a result of that Colombo Gold Centre is housed with 83 jewellery stalls in Pettah, Colombo. However, the global gold prices were dropping in time to time [1] which impacted to local gold market also. Due to this rapid inflation of gold price in global gold market many countries have done the statistical analysis [2,3,4] from different aspects. Therefore, the necessity of statistical analysis for gold prices inflation in Sri Lanka is very important to identify the drop downs to take recovery steps as much as earlier. In [6] has done the statistical modelling for the Sri Lankan gold price volatility and introduced models.

2. MATERIALS AND METHODS

The study is based absolutely on secondary data obtained from various data sources. This secondary data is available in the site www.kitco.com and world gold council website www.worldgold.com. The data of gold price (in US per troy ounce) were collected from 2007 January to 2016 March. US dollar exchange rates for Sri Lankan rupees were taken from Sri Lanka Central Bank website and inflation rates were captured from the site www.goldpinflation.com.

2.1 Preliminary Analysis

At the preliminary stage prior to fit the Auto regressive model, the following techniques were carried out to get an idea about the data and its behaviour.

2.1.1 Plot of Time Series

It is to inspect for extreme observations, missing data, or elements of non-stationary such as trend or seasonality or cyclic pattern or irregular variations.

2.1.2 Augmented Dickey- Fuller test

Augmented Dickey- Fuller (ADF) test is used to test whether the series has a unit root. It is to confirm, statistically, that the stationary of series in terms of trend availability.

The test statistic for the model $y_t = \rho y_{t-1} + u_t$ with $-1 < \rho < 1$, is $DF = \frac{\hat{\rho}}{Se(\hat{\rho})} \sim t_{n-1}$ where y_t is the response variable at time t, u_t is the white noise and n is the number of observations. The hypothesis to be tested in this test is H_0: series is non-stationary $(|\rho| = 1)$ versus H_1: series is stationary $(|\rho| < 1)$.

2.1.3 Kruskal- Wallis test

Kruskal- Wallis test is used to confirm the seasonality in the series. The hypothesis to be tested in this test is, H_0: series has no seasonality versus H_1: series has seasonality.

The test statistic of Kruskal- Wallis test is defined as: $H = \frac{12}{N(N+1)}\sum_{i=1}^{L}\frac{R_i^2}{n_i} - 3(N+1) \sim \chi^2_{L-1}$

where N is the total number of rankings, R_i is the sum of the rankings in a specific season, n_i is the number of the rankings in a specific season and L is the length of the season.

2.1.4 Autocorrelation function and partial auto correlation function

In time series analysis, a process of examining the autocorrelation function (ACF) and partial autocorrelation function (PACF) is to determine the nature of the process under consideration.

2.1.5 Autocorrelation function

Autocorrelation function (ACF) at lag k is defined by

$$\rho_K = \frac{Cov(Y_t, Y_{t+K})}{\sqrt{Var(Y_t)Var(Y_{t+K})}}$$

The first several autocorrelations are persistently large in the graph of ACF and trailed off to zero rather slowly, it can be assumed that a trend exists and the time series is non-stationary. If the series is stationary, then ACF graph must decay exponentially.

2.1.6 Partial autocorrelation function

Partial autocorrelation function (PACF) between is the conditional correlation between Y_t and Y_{t+K} and defined as follows:

$$\phi_{KK} = cor(Y_t, Y_{t+K} \mid Y_{t+1}, Y_{t+2}, Y_{t+3} \ldots Y_{t+K-1})$$

In other words, the PAC between Y_t and Y_{t+K} is the autocorrelation between Y_t and Y_{t+K} after adjusting for $Y_{t+1}, Y_{t+2}, Y_{t+3} \ldots Y_{t+K-1}$.

2.2 Auto Regressive Moving Averages (ARMA) Model

Auto Regressive Moving Averages (ARMA) Model is a statistical model describing the relationship between an output variable Y_t and one or more input variables Y_{t-i}'s and white nose terms.

$$Y_t = \alpha_1 Y_t + \alpha_2 Y_{t-1} + \alpha_3 Y_{t-3} \ldots + \alpha_p Y_{t-p}$$

$$+ \beta_1 e_{t-1} + \beta_2 e_{t2} + \ldots\ldots + \beta_q e_{t-q} + e_t$$

where $e_t \sim N(0, \sigma^2)$

It may be rewritten as $\phi(B)X_t = \theta(B)e_t$ where

$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \ldots\ldots \phi_p B^p$

and $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 - + \theta_3 B^3 \ldots\ldots + \theta_q B^q$ are polynomials in B of degree p and q respectively, and $\{e_t\}$ s a sequence of independent and identically distributed random variables with mean zero and variance σ_e^2.

2.3 Residual Analysis

Before using the model for forecasting, it must be checked for adequacy. Diagnostic checks are performed to determine the adequacy of the model. Accordingly, the residuals should be random and normally distributed with constant variance. The following tests are carried out for the residual analysis:

2.3.1 Anderson- Darling

The Anderson- Darling (AD) test is used to test if a sample of data comes from a population with a specific distribution. It is a modification of Kolmogorov- Smirnov (K-S) test and gives more weight to the tails than does the K-S test. Here the hypotheses are H_0: The data follow normal distribution versus H_1: The data do not follow normal distribution. The test statistic of AD test is:

$$A^2 = -N - \sum_{i=1}^{N}\frac{(2i-1)}{N}\left[\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))\right]$$

where F is the cumulative distribution function of the specified distribution, Y_i are the ordered data and N is the total number of observations.

2.3.2 Lagrange's Multiplier test

Lagrange's Multiplier (LM) test is used to test the independency of residuals. It is an alternative test of Durbin Watson test for auto correlation among residuals. The null hypothesis to be tested is that, H_0: there is no serial correlation of any order. The individual residual autocorrelations should be small. Significant residual autocorrelations at low lags or seasonal lags suggest that the model is inadequate. The test statistic of LM test is:

$W = nR^2 \sim \chi^2_K$ where, K is the number of regressors in the auxiliary regression (only linear terms of the dependent variable are in the auxiliary regression), R^2 is the determination of coefficients and n is the number of observations.

2.3.3 White's General test

White's general test is used in order to check constant variance of residuals. Accordingly the null hypothesis is H_0: Homoscedasticity against the alternative hypothesis H_1: Heteroscedasticity.

Test statistic of White's General test is: $W = nR^2 \sim \chi^2_K$ *where, K is the number of regressors in the auxiliary regression (squared terms of the dependent variable are also included in addition to terms in the LM test in auxiliary regression), R^2 is the determination of coefficients and n is the number of observations.*

Model Validation

It is important to evaluate performance of fitted model on the basis of the fit of the forecasting. Measure of forecast accuracy should always be evaluated as part of a model validation effort.

2. 4 Mean absolute percentage error

Mean absolute percentage error (MAPE) is the average of the sum of the absolute values of the percentage errors. It is generally used for evaluation of the forecast against the validation sample. To compare the average forecast accuracy of different models, MAPE statistics is used. It is defined as, $\mathrm{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \hat{Y}_t}{Y_t}\right| \times 100$ where Y_t is the response variable at time t and n is the number of observations. Practically if MAPE is less than 10% then the fitted model is highly recommended for forecasting.

3. RESULTS AND DISCUSSIONS

TIME SERIES MODELS TO FORECAST GOLD PRICE

This chapter includes the fitted Time Series models to forecast gold price. Preliminary analysis has been carried out to check the validity of the data series for the time series analysis and to understand the behavior of the data series. Under Auto Regressive Integrated Moving Average model building techniques three models were discussed.

3.1Preliminary Analysis

After collecting data it is necessary to test for its suitability to carry on stationary time series analysis.

The Figure 3.1 shows the time series plot for monthly gold price in Sri Lanka which consists of 108 observations.

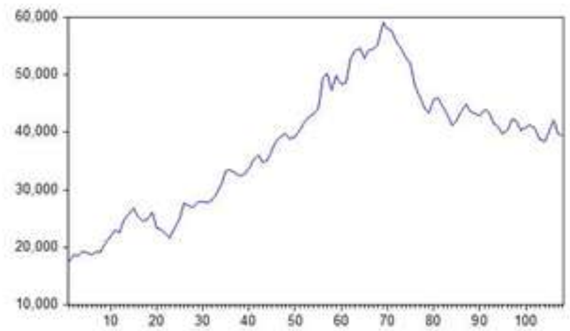
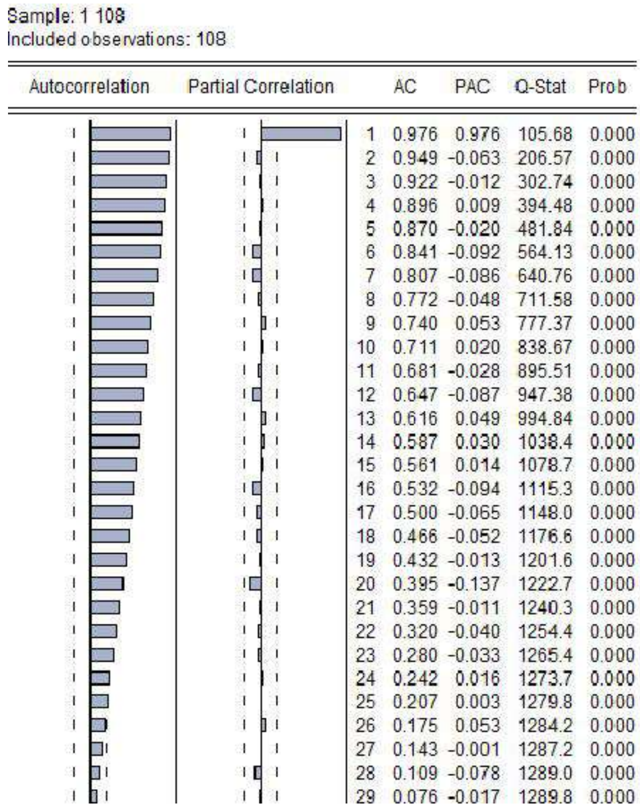


Fig: 3.1: Time series plot of monthly gold price in Sri Lanka from Jan. 2007 to Dec. 2015. The X axis denote the number of months and Y axis denote the gold price(LKR).

Reference to the Fig 3.1, we can see several interesting features. First, data is positively autocorrelated. Second, it exhibits rapid increase which maximized around 2012(62). From 2012(62) onwards there was a significant decline until 2015(108). Further it exhibits small fluctuations but not large swings up and down. However overall trend goes upward which implies that the data is not stationary. Further non stationarity of data series can be justified by autocorrelaiton(ACF) and partial autocorrelaiton(PACF) graphs as well.



	AC	PAC	Q-Stat	Prob
1	0.976	0.976	105.68	0.000
2	0.949	-0.063	206.57	0.000
3	0.922	-0.012	302.74	0.000
4	0.896	0.009	394.48	0.000
5	0.870	-0.020	481.84	0.000
6	0.841	-0.092	564.13	0.000
7	0.807	-0.086	640.76	0.000
8	0.772	-0.048	711.58	0.000
9	0.740	0.053	777.37	0.000
10	0.711	0.020	838.67	0.000
11	0.681	-0.028	895.51	0.000
12	0.647	-0.087	947.38	0.000
13	0.616	0.049	994.84	0.000
14	0.587	0.030	1038.4	0.000
15	0.561	0.014	1078.7	0.000
16	0.532	-0.094	1115.3	0.000
17	0.500	-0.065	1148.0	0.000
18	0.466	-0.052	1176.6	0.000
19	0.432	-0.013	1201.6	0.000
20	0.395	-0.137	1222.7	0.000
21	0.359	-0.011	1240.3	0.000
22	0.320	-0.040	1254.4	0.000
23	0.280	-0.033	1265.4	0.000
24	0.242	0.016	1273.7	0.000
25	0.207	0.003	1279.8	0.000
26	0.175	0.053	1284.2	0.000
27	0.143	-0.001	1287.2	0.000
28	0.109	-0.078	1289.0	0.000
29	0.076	-0.017	1289.8	0.000

Fig. 3.2: Shows the Correlogram of gold price

Together with its sample ACF and PACF are given in Figure 4.2 we can see that the sample ACF dies out very slowly, while the sample PACF is only significant at the first lag. Also note that the PACF value at the first lag is very close to one. All this evidence suggests that the process is not stationary. Consolidated Unit Root Test, Augmented Dickey-Fuller and Phillips-Perron(PP) tests test with intercept to prove data are not been stationary in statistically.

Table 3.1: Shows that the p values for the ADF and PP test statistics

Original Series	ADF t- statistic	PP Adj. t-statistic
	-1.866564	-1.866564
Probability Value	0.3469	0.3469

Table 3.1, p values are given for test critical value of 5% significant level. Depending on hypothesis for ADF and PP tests we can reject the null hypothesis H_0 and accept the alternative hypothesis H_1 said that the data series has no unit root means that the time series data is non stationary. Further there is no special pattern exists in the sample ACF graph means that we can conclude the absences of seasonality in the gold price.

To confirm the stationarity of the first difference series we carried on the ; Unit Root test, Augmented Dickey Fuller and Pillips-Perron tests have been carried out and the results of 5% of significant level are shown the in the Table 3.2.

Table 3.2: ADF test and PP test values for the first difference series.

First difference Series	ADF t- statistic	PP Adj. t-statistic
Probability Value	-8.36265 0.0000	-8.288831 0.0000

Depending on the two tests results ADF and PP test we can conclude the data series is stationary at the 1st difference as the p-value of 1st is difference series is 0:0000 less than the significant level 0.05; the null hypothesis H_0 is rejected and accept the alternative H_1 hypothesis.

3.2.2Model Identification and Coefficient Estimations.

To find the significant AR and MA terms for the suggested model E-views 7 software has been used. The Table 3.3 showed the significance of coefficients ar(3) and ma(3) of ARIMA model . Probability values of two coefficients less than the 5% indicates both values are significant.

Table 3.3 Parameter Estimates of ARIMA model 1 (Eviews output)

Dependant variable d(gold price)

Variable	coefficient	Std error	P value
AR(3)	0.9271	0.01610	0.0000
MA(3)	-0.9370	0.0651	0.0000

R-squared 0.0210 Mean dependant var 195.186
S.E of regression 1507.66 S.D dependant var 1516.34
Akaike criterion 17.49 Schwarz cretrion 17.54
Durbin Wason stat 1.635

Due to the large value of standard error of the fitted model it is 1507.66, we will not recommended as adequate model because most of the estimates will be mislead. Therefore log transformation is used to stabilize the variance. The box cox transformation has been carried out to identify which transformation should be taken into account. Figure 3.3 shows estimated λ value is 0 indicates log transformation is appropriate for god price to stabilize the variance.

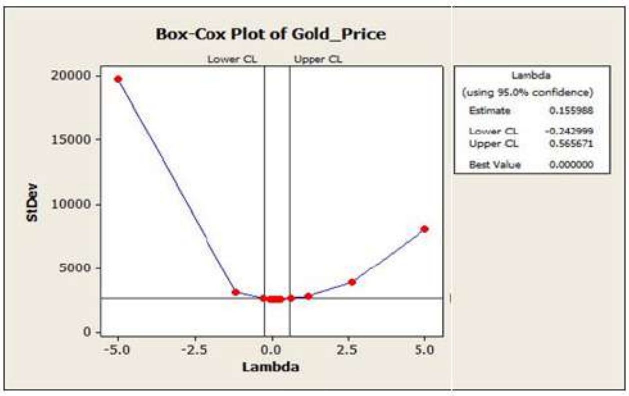


Fig: 3.3: Box Cox transformation plot

Table 3.4 shows ARIMA model 2 for log transformed data series.

Table 3.4 Parameter Estimates of ARIMA model 2 (Eviews output)

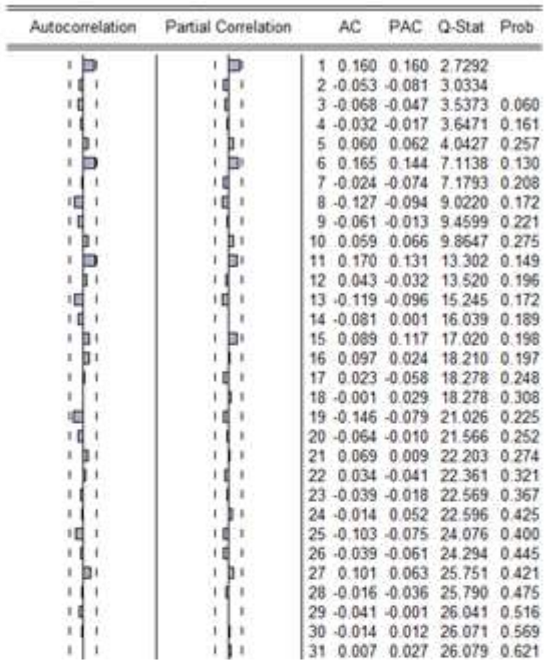
Dependant variable d(log(gold price))

Variable	coefficient	Std error		P value
AR(3)	0.9086	0.04634	19.6058	0.0000
MA(3)	-0.94437	0.04475	-21.1033	0.0000

R-squared 0.0435 Mean dependant var 0.00695
S.E of regression 0.034188 S.D dependant var 0.040180
Akaike criterion -3.6066 Schwarz cretrion -3.5557
Durbin Wason stat 1.67

Residual Analysis of model 2

After finding the significant coefficients residual analysis has been carried out to check the adequacy of the model. The residual sample autocorrelation and partial auto correlation function in Figure 3.4 do not presence any structure and all the values lie between 95% of confidence interval. And their probability values are greater than 5% of significance level means that residuals have nonstationary structure. Thus we can accept the H_0 means that there is no correlation among residuals.



	AC	PAC	Q-Stat	Prob
1	0.160	0.160	2.7292	
2	-0.053	-0.081	3.0334	
3	-0.068	-0.047	3.5373	0.060
4	-0.032	-0.017	3.6471	0.161
5	0.060	0.062	4.0427	0.257
6	0.165	0.144	7.1138	0.130
7	-0.024	-0.074	7.1793	0.208
8	-0.127	-0.094	9.0220	0.172
9	-0.061	-0.013	9.4599	0.221
10	0.059	0.066	9.8647	0.275
11	0.170	0.131	13.302	0.149
12	0.043	-0.032	13.520	0.196
13	-0.119	-0.096	15.245	0.172
14	-0.081	0.001	16.039	0.189
15	0.089	0.117	17.020	0.198
16	0.097	0.024	18.210	0.197
17	0.023	-0.058	18.278	0.248
18	-0.001	0.029	18.278	0.308
19	-0.146	-0.079	21.026	0.225
20	-0.064	-0.010	21.566	0.252
21	0.069	0.009	22.203	0.274
22	0.034	-0.041	22.361	0.321
23	-0.039	-0.018	22.569	0.367
24	-0.014	0.052	22.596	0.425
25	-0.103	-0.075	24.076	0.400
26	-0.039	-0.061	24.294	0.445
27	0.101	0.063	25.751	0.421
28	-0.016	-0.036	25.790	0.475
29	-0.041	-0.001	26.041	0.516
30	-0.014	0.012	26.071	0.569
31	0.007	0.027	26.079	0.621

Fig; 3.4 Sample ACF and PACF values of residuals of Model2

In order to check normality of residuals, Kurtosis, skewness and Jarque-Bera statistic are used. As shown in the Figure 3.5 it is clear that Skewness is 0.01, Kurtosis is 3.39 and Jarque-Bera statistic is 0.683 with P value 0.71 (> 0.05)

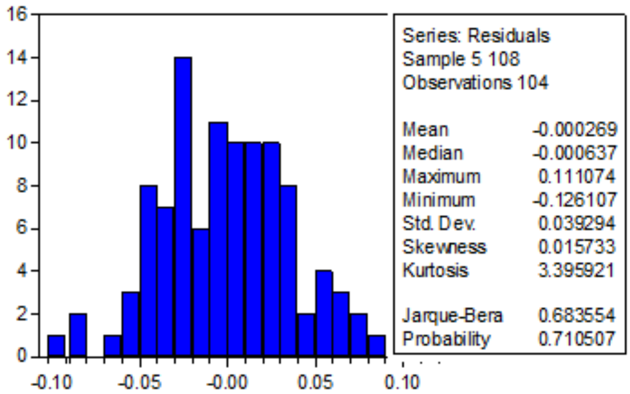


Fig: 3.5 Histogram of residuals of model 2

The Lagrange Multiplier(LM) test has provided a standard means of testing parametric restrictions for a variety of models. Serial Correlation occurs in time-series studies when the errors associated with a given time period.
Figure 3.5 Shows the non Serial Correlation of residuals of model 2 with LM test results of probability values for each term. All probability values are greater than 5% of significant level showed rejecting the alternative hypothesis H_1.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.868126	Probability	0.159762
Obs*R-squared	3.741020	Probability	0.154045

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 04/17/17 Time: 13:53
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(3)	-0.003750	0.046138	-0.081288	0.9354
MA(3)	0.004607	0.044584	0.103341	0.9179
RESID(-1)	0.179282	0.099827	1.795921	0.0755
RESID(-2)	-0.099423	0.100658	-0.987736	0.3257

R-squared	0.035971	Mean dependent var	-0.000269
Adjusted R-squared	0.007050	S.D. dependent var	0.039294
S.E. of regression	0.039155	Akaike info criterion	-3.604858
Sum squared resid	0.153314	Schwarz criterion	-3.503151
Log likelihood	191.4526	Durbin-Watson stat	2.004316

Fig: 3.6 Breusch-Godfrey Serial Correlation LM Test for residuals of model 2

The presence of heteroscedasticity can validate by statistical tests. Here use Whit's general heteroscedasticity test (The White Test). The probability values are greater than the 5% of significance level which means that the null hypothesis H_0 can be accepted. In other words no heteroscedasticity appears among residuals that are they have non constant variance.

ARCH Test:

F-statistic	0.533419	Probability	0.46684
Obs*R-squared	0.541124	Probability	0.461967

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 04/17/17 Time: 13:38
Sample (adjusted): 6 108
Included observations: 103 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001650	0.000281	5.863279	0.0000
RESID^2(-1)	-0.072570	0.099363	-0.730355	0.4669

R-squared	0.005254	Mean dependent var	0.001538
Adjusted R-squared	-0.004595	S.D. dependent var	0.002388
S.E. of regression	0.002394	Akaike info criterion	-9.212568
Sum squared resid	0.000579	Schwarz criterion	-9.161408
Log likelihood	476.4472	F-statistic	0.533419
Durbin-Watson stat	1.994989	Prob(F-statistic)	0.466864

Fig: 3.7 Shows the Heteroscedasticity of residuals of model 2

of gold price.
The careful investigation from the various Tests as mentioned above and Histogram of standard residuals in the fitted model 2 (above in Figure 3.5) infers that standard errors are roughly constant in its mean and variance is constant. To investigate further whether there are any correlations between successive forecast errors, we will plot the correlogram (ACF) and partial correlogram (PACF) of the forecast errors. Following Figure 3.4 represents ACF and PACF of the forecast errors. It is clearly evident from the ACF plot above that none of the autocorrelation coefficients between lag 1 and 20 are breaching the significant limits i.e. all the ACF values are well within the significant bounds.
Fitted model equation

$$(1 - B)(1 - \propto B^3) lnX_t = (1 - \beta B^3) e_t$$
$$lnX_t = lnX_{t-1} + 0.91 lnX_{t-3} - 0.91 lnX_{t-4}$$
$$= 0.94 e_{t-3} + e_t$$

Forecasting and Model Accuracy

Table 4.4: The Forecast gold prices and Error Percentages

Month	Forecast	Actual	Error	Error %
Jan2016	39572	40631	1058	0.026
Feb2016	39635	44424	4788	0.07
Mar 2016	42831	46153	3321	0.071

Future gold prices are estimated using the above fitted model the calculated Mean Absolute Percentage Error (MAPE) is equal to 6.37% for this fitted ARIMA model.

Discussion

A Unit root test was applied to the monthly gold prices it indicates the gold price series is non stationary. After the test of stationary, the test results of ADF and PP test conclude the data series is stationary at the 1st difference.
E-views software is used for fitting the coefficient of the model and after residual analysis the model selected is ARIMA with terms AR(3) and MA(3). Mean Absolute Percentage Error(MAPE) is used to measure forecasting accuracy.

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