Box-Jenkins Approach for Out-of-Sample Forecasting of Stock Price Index: Evidence from Colombo Stock Exchange

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Abstract— *Univariate time series analysis under the time domain has become a widely used data analysis technique from the past. The methods belong to time series could be used to identify the temporal structure of data and forecasting. Among them Box and Jenkins approach has been used in the research studies and could be rendered as the commonly used method. Accordingly, they have introduced a model to predict the future behaviour through an auto-projective method that uses the past behaviour of the series.*

The purpose of this study is to obtain accurate out-of-sample forecasts for stock price indices using an ARIMA model. The daily All Share Price Indices (ASPI) were used over the period from 2nd January 2012 to 31st December 2013 in Colombo Stock Exchange (CSE), Sri Lanka. This study used Box-Jenkins method with four main concepts of model identification, estimation, diagnostic checking and forecasting.

Basically, the concept of information criteria was used for the model identification process. The corresponding model Parameters were estimated at the training data sample using the least square method. Moreover, residual plots and residual tests were used to check the model diagnosis. Finally, different error approaches such as mean absolute error (MAE), root mean square error (RMSE), and mean absolute percent error (MAPE) were used to evaluate the forecast performances of the selected models through different time horizons.

Two models were chosen according to the results of Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and Hannan-Quinn criterion (HQIC.) The out-of-sample forecasting indicates that the selected two models are appropriate for one step ahead forecasting than the long time horizon. The aggregate results depicted that; ARMA (1, 1, 0) is the optimal model and can produce more accurate result than ARIMA (4, 1, 5) for ASPI data within the considered time period.

Keywords— Box-Jenkins method, ARIMA, Forecasting

I. INTRODUCTION

In contrast to the structural models, time series models are different in the model building process by capturing important behaviours of observed data. Beyond that, univariate time series modelling is quite important when the explanatory variables are not available. Among the many methods of forecasting a time series, Box and Jenkins approach has been widely used in research studies and could be rendered as the commonly used method. According to the approach, they introduced a model to predict the future behaviour through an auto-projective method which uses the past behaviour of the series. The corresponding basic model is simply known as ARMA; Autoregressive Moving Average (Box & Jenkins, 1970). It's an important class of model is known as ARIMA; Autoregressive Integrated Moving Average after converting level data into its first difference series.

As a linear statistical model, ARIMA is important to capture the linear features of the past and current observations. According to that, the future value of the series is assumed to be a linear combination of three components: auto-regression (AR), integration (I), and moving average (MA). The corresponding model can be denoted as ARIMA (p, d, q), where p is the number of auto-regression terms, d is the number of non-seasonal differences, and q is the number of lagged forecast errors (Box & Jenkins, 1970; Wang et al., 2012).

Many studies have used ARIMA models to analyse time series data and shown that they obtained the best results comparative to some of the other traditional models. For instance, Ho et al. (1998) investigated repairable system reliability forecasting based on the ARIMA models under the Box-Jenkins methodology. They followed the ARIMA model and the comparison has been made with the traditional Duane model. The corresponding results pointed that ARIMA model is a viable alternative based on its predictive performance. Sharma et al. (2009) adopted Box-Jenkins method for forecasting the ambient air quality data of Delhi City. In their study, several models were developed based on ARIMA and the evaluation statistics showed that the suggested Box-Jenkins approach is satisfactory in the forecasting process. Moreover, they mentioned that the developed models could be used to provide short-term, real-time forecasts of extreme air pollution concentrations.

Jayasinghe and A. Kankanamge (2011) attempted to forecast ASPI using univariate time series techniques with a time trend component. ARMA (1, 1) was selected as the optimal model with the quadratic time trend. Based on one week ahead forecasting results, they showed that a simple ARMA model with a time trend component can be used to produce a reasonably good forecast among many other time series models. In addition Rathnayaka et al. (2014) did a sector vise data analysis on basis of ARMA models. Among the several, ARMA (1, 1) and ARMA (1, 0) were selected as best fitted models for different set of sectors.

Some researchers have concentrated to identify the patterns in the stock prices and forecasting. For example, Herrmann (1980) used the Box-Jenkins approach to determine whether or not the buffer stock policy under the 1980 International Cocoa Agreement stabilized world market prices. Under his methodology, ARIMA models were estimated and used in the forecasting process to fulfil their expectations. Besides, some researchers have attempted to make comparisons of the forecasting ability of ARIMA with the other models. For instance, one study was done by Koch et al. (1994) for forecasting stock returns in Japanese, UK and US markets during the Crash of October 1987. In this case, an ARIMA model was used along with a simultaneous equation model. The empirical results pointed out that neither type of models accurately forecasts the sharp changes in the market indices around the crash. Another comparison was made by Stevenson (2007) by examining ARIMA models in the context of rent forecasting data for the British office markets. Based on the results, the study depicted that the ARIMA models are useful in anticipating broad market trends, but there are substantial differences in the forecast obtained using alternative specifications.

On basis of the previous studies, it can be concluded that an ARIMA model is useful to find the important features of the observed data as well as suitable to make the best forecasts for some sort of time series data. Beyond that, more steps and methods might be required to analyse particular situations. In our study, we attempt to find a suitable ARIMA model for out-of-sample forecasting. The recently published daily All Share Price Indices (ASPI) data are considered over the period from 02nd January 2012 to 31st December 2013. The present study uses Box-Jenkins (1970) method in accordance with four concepts that the model identification, estimation, diagnostic checking and forecasting. Initially, the selected sample is categorized as training and testing according to the ratio 0.85:0.15 and the training data sample is used for the process of model building. Then, the out-of sample forecasting is carried out to all the selected models. Finally, the optimal model is selected based on the performance of mean absolute error (MAE), root mean square error (RMSE), and mean absolute percent error (MAPE).

The paper is organized as follows. Next section briefly describes the basic concepts of the Box-Jenkins methodology for ARIMA forecasting. Section three employs ASPI data to check the appropriateness of ARIMA forecasting through the empirical results. The conclusion and future research directions are rendered in the final section.

II. BOX-JENKINS METHODOLOGY

The methodology consists of four components: model identification, parameter estimation, diagnostic checking and forecasting.

A. *Model Identification*

In the model identification process, an important as well as the initial step is to determine the stationary of the series. If not, the results of the study might be spurious based on the influence of its chaotic behaviours. It can be two types which are trend stationary and difference stationary based on the process of converting a non-stationary series into a stationary series. However, the types of stationarity can be tested by using the behaviour of a correlogram (acf plot), which is based on autocorrelation. As well as unit root tests based on the unit root such as Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS).

1) Autoregressive Integrated Moving Average Model (ARIMA): If a time series does not show any other seasonal or cyclical behaviour, the stationary series can be used to determine the order of the autoregressive terms (AR) and moving average terms (MA). For this, we can use both autocorrelation function (acf) and partial autocorrelation function (pacf) by judging the corresponding acf and pacf plots. In addition, alternative techniques such as information criteria can be used along with the graphical plots. There are mainly three information criteria such as Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and Hannan-Quinn criterion (HQIC).

Based on the Preliminary analysis, if we select a model as ARIMA (p, d, q), then the stationary, invertible, mixed autoregressive moving average process can be written by the equations 1 as follows:

$$v_t = \sum_{i=1}^{p} \phi_i v_{t-i} - \sum \theta_j a_{t-j} + a_t \qquad (1)$$

Where v_t is the d^{th} difference of the original series, $a's$ are independent random variables, $\phi's$ and $\theta's$ are structural constants for the system (Box & Jenkins, 1970; Box & Pierce, 1970).

B. *Parameter Estimation of ARIMA Model*

After specifying the suitable model, Parameter estimation methods can be applied to estimate the Parameters. Several estimation methods are available in the literature such as least square method and maximum likelihood method (Maddala, 2001).

C. *Diagnostic Checking*

The adequacy of the constructed model can be figured out based on the residuals. Initially, the residuals should be checked for the evidence of autocorrelations using its acf and pacf plots for white noise. Then, the tests of residuals are able to use for diagnostics. Under the residual diagnostics, Box-Pierce or Q statistic, Box-Ljung (or Modified Box-Pierce) statistic, Durbin-Waston statistic and Lagrange Multiplier tests have been presented in the theory (Brooks, 2008; Maddala, 2001).

D. *Forecasting Performance*

Forecasting is the final step of the Box-Jenkins method. It can be functioned for one-step ahead or multi-step ahead forecasting horizons. The evaluation of the forecasting values is based on the common performance measures such as mean absolute error (MAE), root mean squared error (RMSE), and mean absolute percent error (MAPE) given by the equations 3, 4 and 5 as follows:

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|Y(t) - \hat{Y}(t)\right| \qquad (3)$$

$$MSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y(t) - \hat{Y}(t))^2} \qquad (4)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|(Y(t) - \hat{Y}(t))/Y(t)\right| \qquad (5)$$

Where $Y_{(t)}$ and $\hat{Y}_{(t)}$ are actual and predicted values at time t, T is the number of observations. Since all the measures are rendered as the deviation between actual and predicted values, the model with the better forecasting results can be chosen based on the minimum values of MAE, MSE and MAPE.

III. EXPERIMENTATION DESIGN AND EMPIRICAL RESULTS

A. Data

The purpose of this study is to find a suitable ARIMA model for out-of-sample forecasting using short time period of historical data. For this, we use recent daily ASPI index in Colombo Stock Exchange (CSE), Sri Lanka over the period from 2nd January 2012 to 31st December 2013. The sample consists of 484 values and the first 411 values (about 85% of the sample) are allocated as in-sample (training sample) and the remaining 73 values are considered as the out-of-sample (testing sample). To enhance the accuracy of the results of the analysis, we use logarithms of the original data series based on the homoscedasticity. The corresponding time series plot of logarithm ASPI (run sequence plot) is shown in Fig. 1.

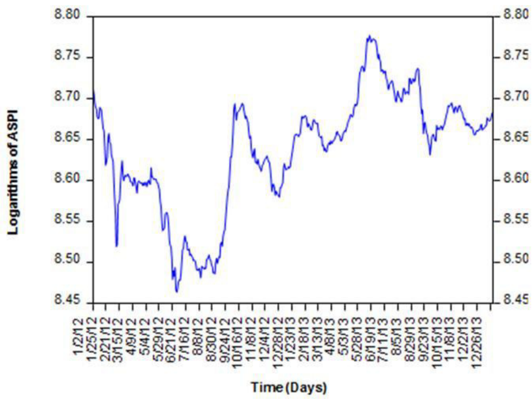


Figure 2. Daily logarithmic data of ASPI

B. Empirical Results

In this study, Eviews 6 and Minitab statistical packages were used in the process of modelling and forecasting. Initially, a run sequence plot was examined to see whether there is any trend or seasonal components in the data series. The Fig. 1 shows the run sequence plot and it reveals that there is not any significant trend or obvious seasonal pattern in the data.

Next, stationary of the series was tested by using acf plot and the unit root tests. The corresponding results are displayed in Fig. 2 below.

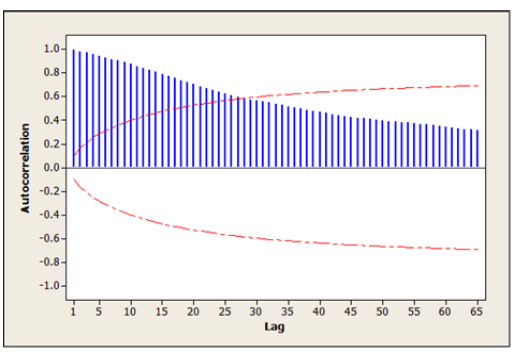
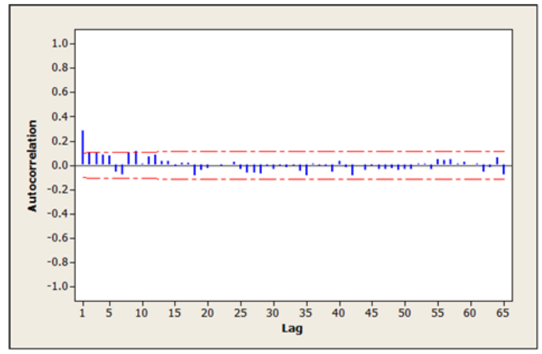


Figure 3. ACF plot of daily logarithmic ASPI data

The acf plot of logarithmic ASPI data depicts that the sample autocorrelations are strong, positive and decaying very slowly. Therefore, it indicates that the series is non-stationary. Moreover, it does not show any periodic behaviour and can be decided that the data set does not have any seasonal component. Hence, we can propose that the ARIMA model is appropriate for the considered data series. Then, the first difference of the original series was obtained and checked the stationary behaviour using *acf* plot which is displayed in Fig. 3.

Figure 4. ACF plot of the 1st difference of daily



logarithmic ASPI data

According to the Fig. 3, we can see that all the autocorrelations lie between the bounds except the autocorrelation at lag 1.

Table 2. Results of unit root tests

Test		**ADF**	
		Intercept	**Trend & Intercept**
ADF (p-value)	Level	0.4512	0.1877
	1st difference	0.0000	0.0000
PP (p-value)	Level	0.3792	0.1432
	1st difference	0.0000	0.0000
KPSS (LM-stat)	Level	1.4271	0.2065
	1st difference	0.1650	0.1116

Based on the ADF and PP test results in Table 2, it can be pointed that the non-rejection of null hypothesis at the level and the rejection of the null hypothesis at the corresponding first difference. Therefore, it can be confirmed that the first difference is required to make the level data into stationary for ASPI series. KPSS test helps decide that the necessity of trend or difference operation to convert the data into stationary series. The corresponding KPSS test result shows the rejection of the null hypothesis of trend stationary for the logarithms of ASPI series. The overall result reveals that the first difference of the series can make the data stationary.

Then, the stationary series was used to identify the suitable models estimated their parameters. Table 3 gives the values of AIC, SBIC and HQIC of the models for ARIMA (0,0) to ARIMA(5,5).

Table 3. Information criteria for ARIMA model selection for daily logarithmic ASPI data

	AIC, SBIC, and HQIC					
p / q	0	1	2	3	4	5
0	-	-6.7994	-6.7981	-6.7974	-6.7961	-6.8036
		-6.7896	-6.7785	-6.7681	-6.7569	-6.7547
		-6.7956	-6.7903	-6.7858	-6.7806	-6.7843
1	-6.8045	-6.8012	-6.8040	-6.7991	-6.7952	-6.7968
	-6.7947	-6.7816	-6.7746	-6.7598	-6.7461	-6.7379
	-6.8007	-6.7934	-6.7924	-6.7835	-6.7757	-6.7735
2	-6.8003	-6.8014	-6.7974	-6.8072	-6.8324	-6.8337
	-6.7807	-6.7719	-6.7580	-6.7580	-6.7735	-6.7648
	-6.7926	-6.7897	-6.7818	-6.7877	-6.8091	-6.8064
3	-6.7985	-6.7904	-6.8059	-6.8004	-6.8065	-6.8139
	-6.7689	-6.7559	-6.7567	-6.7413	-6.7376	-6.7351
	-6.7869	-6.7797	-6.7864	-6.7770	-6.7792	-6.7828
4	-6.7929	-6.7884	-6.8183	-6.8336	-6.8261	-6.8688
	-6.7534	-6.7390	-6.7591	-6.7645	-6.7471	-6.7799
	-6.7772	-6.7688	-6.7949	-6.8063	-6.7948	-6.8337
5	-6.7901	-6.7879	-6.8371	-6.8462	-6.8212	-6.8656
	-6.7406	-6.7286	-6.7679	-6.7671	-6.7322	-6.7668
	-6.7705	-6.7644	-6.8097	-6.8149	-6.7859	-6.8265

Accordingly, two models were selected on basis of the minimum values of information criteria and the significance of the parameters. They are ARIMA (1, 1, 0) and ARIMA (4, 1, 5) respectively.

The adequacy of the selected models was tested through some graphical techniques based on the assumptions of the residuals and some statistical tests. Fig. 4 represents the four plots of residuals based on the ARIMA (4, 1, 5). The normal probability plot and the histogram indicate that an adequate fit of the model provide by the normal distribution. Residual vs. fits plot in Fig. 4: shows that the residuals are well behaved around zero and indicates that the variance of the residuals is equal. The run sequence plot shows that most of the residuals are scattered in the same range and proves that the assumptions of common location and common scale.

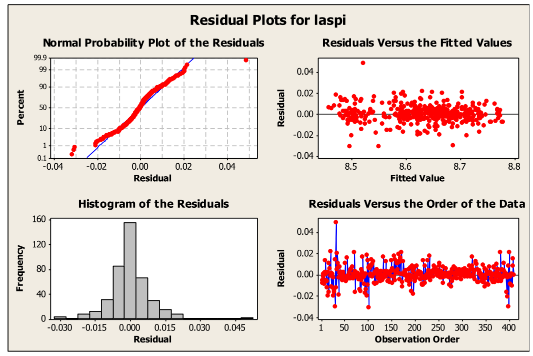


Figure 4. Four plot of residuals for ARIMA (1, 1, 0) model for daily logarithmic ASPI data

In the same way, we examined the adequacy of the remaining model ARIMA (4, 1, 5) as well.

Table 4. Breusch-Godfrey Serial Correlation LM Test for the selected ARIMA models

Model	F-statistic	Prob.
ARIMA (1,1,0)	0.2468	F(1,407) 0.6196
ARIMA (4,1,5)	0.9018	F(5,392) 0.4799

Results in Table 4 indicates that the Breusch-Godfrey serial correlation LM test for the aforesaid two ARIMA models to test the null hypothesis that there is no serial correlation up to specified lag order. For each case, results show that the test does not reject the null hypothesis of no serial correlation up to specific order. As a next step, the selected models can be used for forecasting. Therefore, aforesaid models were used to assess the out-of-sample forecasting performance for the horizon of one day ahead, seven days ahead and 73 days ahead (testing sample). The corresponding RMSE, MAE and MAPE results are summarized in Table 5 below.

Table 5. Performance measures of the selected ARIMA models for daily logarithmic ASPI data

Model	Forecast Period	RMSE	MAE	MAPE
*ARIMA (1,1,0)	1 day	0.8880	0.8880	0.0002
	7 days	41.3129	36.5105	0.0063
	73 days	114.3509	94.2789	0.0160
ARIMA (4,1,5)	1 day	16.8456	16.8456	0.0029
	7 days	57.0805	51.9939	0.0091
	73 days	100.1565	79.5202	0.0135

*denotes the model with the minimum error values

Table 5 shows that, the accuracy of the forecasts decrease when the forecast horizon increases. Accordingly, one step ahead forecast gives more accurate results than the other forecast horizons. Moreover, ARIMA (1, 1, 0) model gives the optimal one day ahead forecast than ARIMA (4, 1, 5).

IV. CONCLUSION

In the present study, Box-Jenkins methodology was used to find the suitable ARIMA model for out-of-sample forecasting using a short period of time. For this, recent two years of daily ASPI data was employed. Basically, the concept of information criterion was used to identify the appropriate models and then the error measures were used to find the better forecasting results.

Based on the results, we can suggest that the two ARIMA models are more suitable for one step ahead forecasting. This result is similar to the conclusion of the study (Sharma et al., 2009) and they also pointed that the ARIMA models could be used to provide short-term forecasting. The aggregate results of our study depicted that; ARMA (1, 1, 0) is the optimal and can produce the most accurate result than ARIMA (4, 1, 5). Based on the overall result, it can be suggested that the used Box-Jenkins approach is sufficient for obtaining satisfactory short term forecast.

The conventional ARIMA model can effectively capture the linear components of the series. Generally, stock prices are chaotic and show both linear and nonlinear behaviours. Therefore, the accuracy of forecast might be enhanced by modelling the non-linear behaviours of the series as well. Therefore, further modifications could be included to the selected model to enhance the accuracy of the forecast. Further, the accuracy of ARIMA forecasting can be evaluated by comparing with forecasting results of other linear and non-linear models.

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