A Review on Geometric Brownian Motion for Forecasting the Daily Share Prices in CSE

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Abstract— *Capital Investments in stock market is the easiest and fastest way for building healthy financial foundation for our future life. As a result, stock markets have become more institutionalized in the past few decades and have been advanced as the main forms of investments for numerous organizations as well as individuals for arranging large investment funds to the general public. Present study mainly focuses an attempt to develop a suitable model based on Geometric Brownian Motion (GBM) to estimate the short-term share price behaviours in the Colombo Stock Exchange (CSE), Sri Lanka based on daily price indices data from January to December, 2013. Furthermore, a time series technique such as ARIMA was used to compare the results.*

Keywords— GBM, ARIMA and MAPE

I. INTRODUCTION

Data analysing and forecasting in the financial sector has been regarded as one of the biggest challenges in the modern economy today. Especially, economical decisions should be taken for predicting future results might involve numerous economic policies as well as economical behaviours.

During the last few decades, equality markets all over the World have been advanced as main forms of investment for numerous organizations and individuals arraying for large investments funds. As a result, huge companies as well as cooperation's have been offering shares to general public and raise their money needed for restructuring, expansion for new operations. It is common phenomenon that, if the company has obtained the capital needed, the shareholders will take benefit through dividends paid by companies. So, the shareholders have alternatives for transferring or selling their ownerships as their requirements. When the company is running well, the price of their stocks would also be going up and make a profit for the investors.

According to the previous studies, stability of the stock market and the economic growth and development of the country are highly associated to one another. Furthermore, Government rules and regulations, numerous macro-economic and micro economic conditions, political and financial stability of the country are also significant predictors of the volatility of the market fluctuations.

Balaban et.al (1996) have examined the informational efficiency of the Turkish Stock Market by using the daily closing prices of equally weighted composite index from 1985 to 1995. Various macro-economic variables such as interest rates, money supply aggregates and currency rates were used as predators of the information effeminacy of the market. Their findings suggested that interest rates and money supply aggregates have significant influence over the market fluctuations. Poshakwale et al. (1998) also conducted a similar type of case study to evaluate the market efficiency in the Bombay Stock Exchange within a six-year period from 1987 to 1994. Non parametric techniques were deeply applied for evaluating the skewness properties of the frequency distributions in the price levels of BSE. The results indicated that Bombay market (BSE) was not in a weak form efficient.

The unbalanced behaviors pose a high risk for the investments. However, higher risk always brings higher returns. The difference between risk and return indicates whether the investment is good or

bad (Tan, 2012). Gupta et al. (2007) attempted to find out the short-run and long-runs relationships between stock market validity with respect to the macro and macro-economic variables. Randomly selected six market indices and related macro factors such as inflation, exchange rates, money supply and interest rates were used. Study result shows the strong co-relation between macroeconomic variables on the stock market indices as well as stock's volumes (Campbell, 1997).

Colombo Stock Exchange (CSE) is one of the most modernized stock exchange in South Asia with a fully automated trading platform. Moreover, it has been one of the best preform stock exchange in the world with market capitalization over US$23 billion with average daily turnover rising over US$18 billion. Because of the political stability of the country, many local and foreign investors have been investing huge capitals after the end the civil war in the north part of country in 2009 (Samarathunga, 2008). However, comparing with other stock exchanges, very few studies can be seen in relevance to the CSE.

Using the market and sector indices, Samarakoon et.al (1996) has investigated the market behaviors of the short term (daily, weekly, monthly and quarterly) predictability of the stock returns with respect to the inflation rates in Sri Lanka from 1985 to 1995. According to their study, inflation rates can be categorized in to four categories namely contemporaneous, lagged, expected and unexpected inflations. The results suggest that both lagged inflation and expected inflation are significantly and positively correlated with stock returns. However, unexpected inflations are found to be negatively related to stock returns.

A review of extant literature suggests that more empirical studies are required to investigate the behaviour of stock prices. Athapathu et al. (2011) has done a similar type of study and examined the relationship between the stock market performance and economic growth using the data for the period spanning from 1997 to 2008. Results of Granger causality and Vector Error correction reveal that there is a long run equilibrium relationship between the economic growth and stock market performance in Sri Lanka (abeysekara, 2001).

There are Two price indices mainly manipulation in the CSE namely All Share Price Index (ASPI) and the S&P Sri Lanka 20 Price Index (S&P SL20) (Rathnayaka, 2013). The S&P SL20 is the principal stock index, which is computed by using the best performing 20 companies' stock prices. This study basically aims to understand the trends and cyclic patterns in the CSE in order to predict the future behaviors.

The rest of the paper is organized as follows. Next section briefly explains the methodological approach used in the study and Section three presents the outcome of statistical analysis and discusses the result of the study. Final section concludes the study alone with proposing avenues for future research

II. METHODOLOGY

The study was carried out on the basis of secondary data, which were obtained from CSE, annual reports of Central Bank of Sri Lanka and different types of background readings and other relevant sources. Daily trading data (more than 240 daily observations) from January 2013 to 2013 December were extracted and tabulated.

Data analysis consists of several phases. In the first phase, stock market validations were identified using traditional auto regressive methods such as ARMA and ARIMA models. In the second stage, proposed GBM algorithm was used to predict future predictions. Finally, similarities and discrepancies between the traditional time series approaches and our proposed model was discussed based on the MAD and MAPE testing accuracies techniques.

F. *ARIMA Model Approach*

The Autoregressive integrated moving average (ARIMA) model has been successfully applied today in wide areas for predicting future movements in the non-stationary data patterns. ARIMA model is a generalization of an Autoregressive moving average model. Basically, it consists three parts. They are; the auto regressive parameter (p), the number of differencing passes (d) and moving average parameter (q) (Seneviratne, 2013; Breckling, 1989).

The moving average process in equation 01 and auto regressive process in equation 02 can be written as follows ;

$$X_t = \mu + Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots,\ldots + \theta_q Z_{t-q} \qquad (1)$$

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + ...,... + \theta_p X_{t-p} + Z_t \quad (2)$$

Where; $Z_t \sim WN(0, \sigma^2)$ and $\theta_i, \vartheta_i \, and \, \mu$ represent the constants. Considering properties of AR and MA processes, ARMA (p, q) can be written as follows;

$$\varphi(B) X_t = \theta(B) Z_t \quad (3)$$

Where; $\varphi(Z) = 1 - \varphi_1 Z - ... ... - \varphi_1 Z^p$ and $\theta(Z) = 1 + \theta_1 Z + ... + \varphi_q Z^q$.

The likelihood estimation method is used for identification and estimation patterns. Autoregressive Integrated Moving Average (ARIMA) models are an extension of ARMA process by intergrade (I) part. ARIMA model is generally referred to as ARIMA (p, d, q), where p, d and q represent the order of auto regressive process (AR (p)), integrated (I(d)) and moving average process (MA (q)) respectively (Jaggia, 2010, Wang 2008).

A. Geometric Brownian Motion

A geometric Brownian motion is a continues- time stochastic process, which satisfies a stochastic differential equation follows a Brownian motion with drift. Currently, it is wildly using for the dynamic structural models to provide a more realistic approach in the financial sectors.

The drift rate or growth rate of the distribution can be assumed as a constant and defined as equation 04 (Kong, 2010).

$$\text{Mean} = \mu \delta t \quad (4)$$

The volatility (standard deviation) of the distribution is significant and elusive quantity in the theory of derivatives. The standard deviation of the asset returns over a time step δt is given as equation (05).

$$\text{Standard Deviation} = \sigma \delta t^{1/2} \quad (5)$$

Putting these scaling's explicitly from equation 04 and equation 05 into asset return model as follows in equation 06.

$$R_t = \frac{S_{t+1} - S_t}{S_t} = \mu \delta t + \sigma \delta t^{1/2} \quad (6)$$

The term dw_t be a random variable, from normally distributions with mean zero and variance δt . So equation (06) can be simplified as follows.

$$dS_t = \mu S_t \, \delta t + \sigma S_t \, dw_t \quad (7)$$

Integrating equation (7) with respect to t;

$$\int_o^t \frac{dS_t}{S_t} = \mu \, t + \sigma \, w_t \quad (8)$$

Where; let we assume that, $w_0 = 0$. The term dS_t / S_t coincided to the derivative of $\ln S_t$. It is clear that, S_t term under the Ito process. So we used Ito's calculation for our further study.

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma w_t\right) \quad (9)$$

Where; $w_t = x_t - x_0$. Equation (09) indicated continuous stochastic process of Geometric Brownian motion that we used for simulated the forecast of stock market indices.

III. RESULTS AND DISCUSSION

The study was carried out on the basis of two hundred and fifty (240) daily trading data for 12 months period from January 2013 to December 2013.

B. Stationary/Non Stationary Model Checking

Table 1. ADF AND PP Test Results

Veriable	p-value associated with the test		*Veriable*	p-value associated with the test	
	Level data			*1st Difference*	
	ADF Test	PP TEST		*ADF Test*	*PP Test*
SAP	*0.575*	0.553	SAP	0.047	*0.043*
ASPI	*0.375*	0.387	ASPI	0.032	*0.037*

The Unit root test results in Table 1 suggested that, two series of data are stationary at only their 1st differences. So that Table 1 result suggested that, ARIMA models are the most significant and suitable for predicting future patterns. The minimum values of Akaike information criterion (-9.063916 and -8.965017) and Schwarz criterion (-8.993680 and -8.932374) suggested that, ARIMA (3, 2, 1) and ARIMA (1, 1, 1) models were most suitable for

predicting future patterns of ASPI and SAP respectively.

Table 2. Forecasting Results: ASPI

Dates (2013)	ASPI				
	Actual Values	ARIMA (3,1,2)	Residual	GBM	Residual
2 / 12	5774.1	5778.333	-4.23301	5775.09	-0.99
3/ 12	5773.12	5778.171	-5.05082	5774.1	-0.98
4/ 12	5782.92	5773.992	8.92762	5779.12	3.8
5/ 12	5780.58	5782.96	-2.37962	5782.92	-2.34
6/ 12	5810.24	5784.109	26.1307	5799.58	10.66
9/ 12	5821.3	5815.916	5.38385	5820.24	1.06
10/ 12	5778.52	5823.213	-44.6932	5782.3	-3.78
11/ 12	5787.81	5777.016	10.7942	5788.52	-0.71
12/ 12	5793.19	5785.472	7.71754	5789.81	3.38
13/ 12	5795.66	5792.56	3.09982	5793.19	2.47
17/ 12	5811.88	5804.433	7.44728	5815.66	-3.78
18/ 12	5811.76	5810.181	1.57897	5811.88	-0.12
19/ 12	5867.00	5814.031	52.9686	5861.76	5.24
20/ 12	5857.36	5881.598	-24.2376	5861.1	-3.74
23/ 12	5846.42	5853.232	-6.81201	5847.36	-0.94
24/ 12	5845.93	5851.719	-5.78936	5846.42	-0.49
26/ 12	5848.49	5841.222	7.26788	5845.93	2.56
27/ 12	5876.66	5850.191	26.4687	5878.49	-1.83
30/ 12	5899.2	5886.164	13.0364	5896.66	2.54
31/ 12	5912.78	5901.099	11.6813	5909.2	3.58
MAPE (%)		**0.236**		**0.047**	
MAD		**13.785**		**2.7495**	

Table 3. Forecasting Results: SAP

Dates (2013)	SAP				
	Actual Values	ARIMA (1,1,1)	Residual	GBM	Residual
2 / 12	3174.63	3174.223	-8.34272	3166.63	0.03
3/ 12	3165.88	3164.691	-3.75091	3160.88	-1.01
4/ 12	3160.94	3159.549	5.50052	3163.94	-1.00
5/ 12	3165.05	3164.439	-2.8191	3165.05	0.00
6/ 12	3161.62	3160.824	22.26554	3179.62	2.01
9/ 12	3183.09	3184.72	9.57968	3194.09	-1.01
10/ 12	3194.3	3196.716	-20.0963	3179.3	1.00
11/ 12	3176.62	3176.656	5.61381	3179.62	-3.00
12/ 12	3182.27	3182.902	6.53806	3188.27	0.00
13/ 12	3189.44	3190.681	-5.0113	3187.44	2.00
17/ 12	3185.67	3186.247	7.62313	3192.67	0.00
18/ 12	3193.87	3195.176	-3.4656	3193.87	0.00
19/ 12	3191.71	3192.502	31.95834	3223.71	2.01
20/ 12	3224.46	3228.459	-0.28891	3227.46	-3.00
23/ 12	3228.17	3231.613	-7.37322	3224.17	1.00
24/ 12	3224.24	3226.474	-1.34358	3224.24	.000
26/ 12	3225.13	3226.941	-3.82114	3225.13	0.00
27/ 12	3223.12	3224.311	16.31873	3239.12	0.00
30/ 12	3240.63	3243.362	24.08805	3268.63	-1.01
31/ 12	3267.45	3272.312	-8.44217	3263.45	-1.00
MAPE (%)			**0.0517**		**0.029**
MAD			**1.6648**		**0.951**

The forecasting result's with error analysis results in Table 2 and Table 3 show that, proposed GMB and ARIMA models are highly accurate (less than 10%) with lowest MAPE error values.

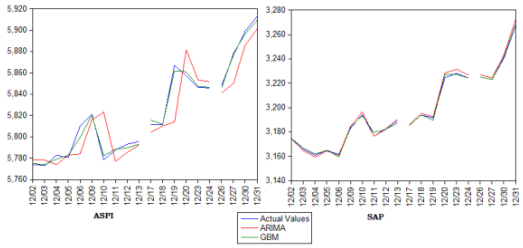

Figure 1. ASPI and SAP forecasted results

Moreover, results clearly show that proposed GBM forecasting model (GBM- ASPI: 0.047%, GBM_SAP: 0.029%) is more significant than traditional ARIMA methods (ARIMA(3,2,1): 0.236%, ARIMA (1,1,1) : 0.0517%) for forecasting short time predictions. According to the Figure 1 ASPI and SAP plots also suggested that, proposed GBM model is more suitable for forecasting stock market indices than tradition forecasting approaches.

IV. CONCLUSIONS

Behavioral analysis of the stock market fluctuations are an important research direction today. Miscellaneous kind of research works based on financial markets around the world can be seen in the literature. The current study is mainly deals with the design and methodology used in explaining the predictive ability and profitability of technical trading strategies in the Colombo Stock Exchange, Sri Lanka.

In the past few decades research works show that, time series approaches such as ARMA and ARIMA were widely used for data forecasting. However after 2000, computational algorithms have been successfully applied to real-world problems. As a result, different type of computational methods such as Geometric Brownian Motion, Neural network models have been developed.

In the current study, two types of statistical techniques were widely used to discuss our result.

They are: GBM and ARIMA approach. The minimum values of Akaike information criterion and Schwarz criterion suggested that, ARIMA (3, 2, 1) and ARIMA (1, 1, 1) models are most suitable for predicting short-term patterns of ASPI and SAP price indices respectively. Furthermore, the mean absolute percentage error (MAPE) result reveal that (ARIMA (3, 2, 1) : 0.00236 and GBM :0.00047/ ARIMA (1, 1, 1) : 0.000517 and GBM : 0.00029), this new proposed model is more significant and gives the best solution to predict short term predictions in high volatility fluctuations.

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