On an Algorithm to Prove the Strong Goldbach Conjecture

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Abstract— *The strong Goldbach conjecture says that any even number greater than the integer 2 can be expressed as the sum of two prime numbers which has been numerically justified up to such large numbers that of order 4 $.10^{18}$. We assume the conjecture is true for all numbers and provide simple algorithm to prove the conjecture for a special class of even numbers first. Secondly, we point how the conjecture may be proved in general.*

Keywords— *strong conjecture, prime numbers, even number*

I. INTRODUCTION

Many authors have attempted to prove the conjecture and for the sake of brevity, we have included in the reference, the most relevant contributors' work. In this paper, we assume that the conjecture is true for all even numbers and in this; we first provide a proof for a special class of even numbers.

Many authors have contributed to Goldbach conjecture and therefore we had to include only the most relevant references in this article.

II. METHODOLOGY AND PROOF

First, we prove that the number of prime numbers in the interval $(0, n]$ and in the interval $(n, 2n)$ are roughly the same for large. In this respect, the prime number theorem is used.

Proof:

Number of primes between 0 and n is roughly $\frac{n}{logn}$ for large n by the prime number theorem.

Number of primes between 0 and 2n is $\frac{2n}{log2n}$ by the prime number theorem.

Therefore number of primes between n and 2n is $\frac{2n}{log2n} - \frac{n}{logn} = \frac{2n}{log2+logn} - \frac{n}{logn} \approx \frac{n}{logn}$ for large n .

Also, we note that $\frac{n}{logn}$ tends to infinity as n tends to infinity.

Proof of the conjecture for positive numbers of the given form

Step.1

If the even number is $2n$ where n is a prime, then $2n = n + n$ and the conjecture is trivially true.

First, we note that number of primes is infinite and therefore we can choose a prime number greater than any given positive integer.

Step.2

Now, let us consider the even number of the form N_2 =2. $2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s}$, where N is a sufficiently large prime number where $t_i \geq 1, i = 1,2,..s$. Recall that if the even number $N_1 = 2^{t_1}.3^{t_2}.5^{t_3}\ldots N$ is large enough, by the above proved result, prime number theorem, we can choose a prime p_1 greater than

$$N_1=2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s},$$

and so close to the number N_2 and less than it and not being equal to $N_2 - 1$ .Note that $N_2 - 1$ may be a prime. Let $k = p_1 - 2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s}$, Note that k is not divisible by any prime number in N_1.

Now, $2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s} - k = p_2$ must be a prime.

Therefore, $p_1 + p_2 = 2.2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s}$. Why we choose p_1 so close to the number N_2 is that then $2^{t_1}.3^{t_2}.5^{t_3}\ldots N^{t_s} - k$ is small and definitely be a prime number. Also, it is not divisible by any prime $2,3,5,7,\ldots,N$

III. NUMERICAL EXAMPLES

(1)$2.3..5 + 23 = 53$, $2.3.5 - 23 = 7$,
$2.2.3.5 = 53 + 7$=60,$k = 23$, $p_1 = 53, p_2 = 7$
(2)$159937 - 2^3.3^2.5^3.7 = 2063 = k$
$2^3.3^2.5^3.7 - (159937 - 2^3.3^2.5^3.7) = 2063$,
$2063 + 159937 = 2.2^3.3^2.5^3.7, p_1 = 159937, p_2 = 2063$

(3) 2.3.5.7.11.13.17.19.23+223092833=446185703
2.3.5.7.11.13.17.19.23-223092833=37,
37+446185703=2.2.3.5.7.11.13.17.19.23

(2.3.5.7.11.13.17.19.23=223092833)
$k = 223092833, p_1 = 446185703, p_2 = 37$

Step.3

Now, consider the even number of the form $N_2 = 2.3.5.7\ldots N$

Our $N_1 = 3.5.7\ldots N$.As before, we choose the prime number (p_1) so close to N_2 but not being equal to $N_2 - 1$.

We define $k = p_1 - 3.5.7\ldots N$, Then $3.5.7\ldots N - k = p_2$ and

$p_1 + p_2 = 2.3.5.7 \ldots N$

<u>Numerical Examples</u>

(1)3.5+8=23, 7+23=2.3.5=30, $p_1 = 23$, $k = 8, p_2 = 7$

(2)$2.3^2.5^2.7^2$=22039+11 $p_1 = 22039$, $k = 11014$, $p_2 = 11$

(3)3.5.7.11.13+14998=30013, 17+30013=2.3.5.7.11.13 , $p_1 = 30013$, $k = 14998$

IV. DISCUSSION AND CONCLUSION

Consider the interval [n 2n] where n is any positive number greater than or equal to four.

Then at least one of 2n-3, 2n-5, 2n-7. . .2n-N is a prime where N is the biggest prime such that 2n-N $\geq n$. This is the reason that we choose $N_1 - k$ above to make a prime.

The above claim was checked numerically and found to be true. For example, in case of $n = 400$, we have found that there were set of 15 elements satisfying

$2n - p =$Prime. In case of $n = 500$, we have found that 24 elements satisfy the above condition. In case of the odd number $n = 551$, the above number decreased to 21, however. In case of $n = 1000$, the above number increased up to 28.In general, our claim that at least one of 2n-3, 2n-5, 2n-7. . .2n-N is a prime. where N is the biggest prime such that 2n-N $\geq n$,is satisfied for all numbers we have examined. In case of all numbers we have examined, we found that any even number $2n = n - k + n + k$ where $n - k$ and $n + k$ are primes for integer. In other words prime distribution in both sides of any integer is roughly symmetric. For example, $1000 = (500 - 387) + (500 + 387)$ and in case of $2n = 200000$, for example, we have 200000 = (100000+9297) + (100000-9297).

REFERENCES

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