Using Markov Processes for Efficient Project Management

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Abstract
An important requirement in project management is to complete it satisfactorily and within the allocated time. Delays incurred during the project will not only result in cost escalation but also in losses benefits that the end result of the project will provide.

Critical Analysis Method (CPM) or Program Evaluation & Review Technique (PERT), are methodologies that identify the critical activities that cannot be delayed as the delay of such will delay the duration of the project.

Accordingly, every effort has to be made to reduce such delays and the only method available to reduce such delays is to use resources in the form of material, equipment and labor efficiently. As such there does not appear in the available literature on techniques that are based on known scientific principles to bring such delays to the minimal level.

The objective of this research is to use Markov process method to forecast the probability of delay in the work in the period in the following time period, based on data on delay or no delay. For convenience a day is taken as the time period, (suitable for minor projects) and activities on the critical path are treated daily. The Markov process calculations can start only after a lapse of a few days from the start which can be 5 or 10 days and the number can be increased From the sequence of delay (D) and no delay (N) up to the current period the Markov stochastic matrix A is constructed which when applied to the vector of current data will yield the probabilities for D and N for the following day. At any current period, if the forecast for the following period is delay, then extra precautions should be taken in implementing the project in the following time period.

Keywords—
Project management, CPM, PERT, Markov Chain Process

I. INTRODUCTION

Critical Path Method or Analysis (CPM) or its counterpart Program Evaluation and Review Technique (PERT) had been widely used especially in planning and scheduling, (Armstronh,1969), management of projects especially in civil construction, (Hendrickson et.al. 2008), but also in areas such as production, (Blake et.al. 2014).

The text book “Operations Research: An Introduction” (Taha, 2002), is equipped with a CD containing software for CPM.

In spite a wide variety of application areas, CPM or PERT does not appear to have been applied to minimize delays of projects except for giving information on critical activities and allowable delay or slack times for non-critical activities. However, critical chain management is seen to be applied in supply change situations, (Goldrat, 1997), (Leach, 1999), (Herroelen, 2002), which somewhat deferrer from CPM.

This research is conducted purely for forecasting probabilities of delays or otherwise for critical activities so that more attention can be brought to such activities in a formal quantitative manner.

A method due to Markov, a stochastic process is to be used for this purpose. It is seen that Markov processes have been applied to make probability forecasts of a variety of events ranging from biology, medicine and even cricket, (Cambridge Statistical Laboratory, 2002), Nielsen et. al., 2015), (McGilchrist, et. al., 1989), (Beck et. al, 1983), (Colwell et. al., 1995).

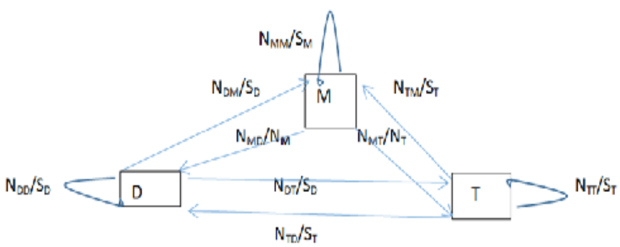
Based on past data on possible outcome of daily events such as delay or otherwise of work on critical activities a Markov stochastic matrix is computed. Then using such data of the current day, forecasts of probabilities of outcomes are calculated. This information will be vital for the management to tackle work on the critical activities for the next few days.

The method of approach merging CPM and Markov process will be outlined in section 2, with some results in section 3, and finally with conclusions in section 4.

II. Method used in the Research

CPM analysis is conducted for the project of interest by identifying the activities, duration of each and the logical connection between activities. The outcome will be identifying the critical activities and then the total project duration. Work that has to be done for each such activity has to be divided to what is to be done on a daily basis.

Let the outcome in terms of work done on a day is to be classified as delay (D), moderate delay (M), or in time no delay (T). The sequence of outcomes is taken as x_i, i =0, 1, ..., n; the nth day being the one before the current day. Let the total numbers of the three groups be S_D, S_M and S_T , respectively. Then the numbers of each type on a particular day followed by that of the following day can be represented as N_{pq} where p and q can be one of D, M, or N. Thus the transitional probabilities can be as shown in the following figure 1.



Accordingly, the Markov stochastic matrix A, in the order N, M, and T; is given by

$$A = \begin{bmatrix} N_{DD}/S_D & N_{DM}/S_D & N_{DT}/S_D \\ N_{MD}/N_M & N_{MM}/S_M & N_{MT}/N_T \\ N_{TD}/S_T & N_{TM}/S_T & N_{TT}/S_T \end{bmatrix}$$

If the current day the status of the work is M, then the current vector on day 0 is v_0 = [0 1 0]' then, v_1 = A' v_0

As such as vector on day n is given by $v_n = (A')^n v_0$

Thus the probability of N. M. T on nth day is 1st, 2nd, 3rd component of v_n, respectively. If the probability of N or M is very high then extra or moderately high precaution has to be taken in implementing work on the nth day. The matrix A has to be updated regularly and the procedure adjusted accordingly.

III. Results of a Simple Application

Starting activities with duration times (in days) and the corresponding critical path for a project of construction of a simple house is as shown in Figure 2.

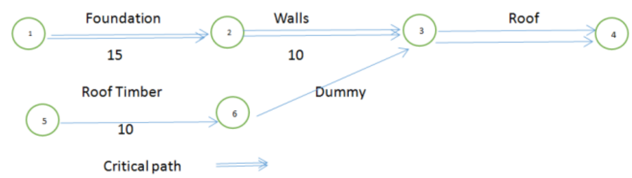


Figure 2 – Critical path for a simple project

The sequence of the status of work for each of the first 20 days is shown below:

D M T T M, M T T D D, M T T D M, D T M T M

Hence S_D =5, S_M = 6 and S_T = 8

N_{DD} = 1, N_{DM} = 3, N_{DT} = 1, N_{MD} = 1, N_{MM} = 1, N_{MT} = 4, N_{TD} = 2 , N_{TM} = 3, N_{TT} = 3

Hence

$$A = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{6} & \frac{1}{6} & \frac{4}{6} \\ \frac{2}{8} & \frac{3}{8} & \frac{3}{8} \end{bmatrix} \text{ and } A' = \begin{bmatrix} \frac{1}{5} & \frac{1}{6} & \frac{2}{8} \\ \frac{3}{5} & \frac{1}{6} & \frac{3}{8} \\ \frac{1}{5} & \frac{4}{6} & \frac{3}{8} \end{bmatrix}$$

$$A = \begin{bmatrix} & & N_{DT}/S_D \\ /N_M & /S_M & N_{MT}/N_T \\ N_{TD}/S_T & N_{TM}/S_T & N_{TT}/S_T \end{bmatrix}$$

V_{20}' = [0 1 0]; $v_{21} = A'.\, v_2$ and v_{21}' = [1/6 1/6 4/6]

Thus most probable status is T , i.e. work on day 21 will be completed in time.

Also $v_{22} = (A')^2 v_{20}$ and v'_{22} = [0.228 0.378 0.394] giving T as the probable outcome. However, the actual outcome could be different from the one forecasted. After a few days the matrix A can be updated including extra information available.

IV. Conclusion

Forecasting future events will not always provide realistic results. An example is to forecast results of cricket matches that had been attempted, (Colwell et. al, 1995), based on results of previously held matches between the same two teams.

However, in the matter of project management such techniques have to be tried out in order to minimize delays in the completion of projects. As mentioned earlier, it is always proper to use modelling and quantitative techniques to supplement strategies based on common sense. In addition to other benefits such techniques help to uplift morals and motivation of the fellow workers.

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